Massachusetts
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General Curriculum
(103/203)

PRACTICE TEST APPENDIX:
Multiple-Choice Question Analyses

Version 2
BOOKLET 2
Mathematics Subtest
(203)

Evaluation Systems, Pearson, P.O. Box 226, Amherst, MA 01004

06/16

MULTIPLE-CHOICE QUESTION ANALYSES

1. A researcher estimated that there were eight billion grains of sand per cubic meter on one particular beach. If the mass of one of these grains of sand is approximately 3.5×10^{-7} grams, what is the approximate mass of one cubic meter of sand on this particular beach?

   A. 2.8×10^{0} grams

   B. 2.8×10^{1} grams

   C. 2.8×10^{2} grams

   D. 2.8×10^{3} grams

Correct Response D: Scientific notation is a way to write and compute with very large and very small numbers. Standard form of scientific notation is always a number between one and ten (including one but not 10) multiplied by a power of ten. Recall that negative exponents represent fractions/decimals. For example, $10^{-1} = \frac{1}{10^{1}} = \frac{1}{10} = 0.1$ and $10^{-3} = \frac{1}{10^{3}} = \frac{1}{1000} = 0.001$.

Eight billion is an 8 followed by nine zeros, or 8×10^{9}. Each one of those grains of sand weighs 3.5×10^{-7} grams. 3.5×10^{-7} needs to be added 8 billion times, but repeated addition leads to multiplication, so the result is found using $(8 \times 10^{9}) \times (3.5 \times 10^{-7})$. Multiplication is both commutative and associative, so the order and grouping can be rearranged to make it more convenient.

$(8 \times 10^{9}) \times (3.5 \times 10^{-7})$ can be rewritten as $(8 \times 3.5) \times (10^{9} \times 10^{-7}) = 28 \times 10^{9+(-7)} = 28 \times 10^{2}$, but 28×10^{2} is not in the standard form of scientific notation. $28 = 2.8 \times 10^{1}$ which now gives a numeral between one and ten. $28 \times 10^{2} = 2.8 \times 10^{1} \times 10^{2} = 2.8 \times 10^{1+2} = 2.8 \times 10^{3}$.

Incorrect Response A: This response was probably a conversion error. Eight billion was converted into 8×10^{6} which is 8 million. The computation was done correctly from there.

Incorrect Response B: This response adjusted the exponent in the wrong direction, making 28 into 2.8×10^{-1} instead of 2.8×10^{1}.

Incorrect Response C: This response results from changing 28 into 2.8 but making no adjustment in the size of the exponent. Since $28 \neq 2.8$, the final result is incorrect.

2. If *k* represents an irrational number, which of the following operations must always result in an irrational number?

   A. $k + k$

   B. $k – k$

   C. $k \times k$

   D. $k \div k$

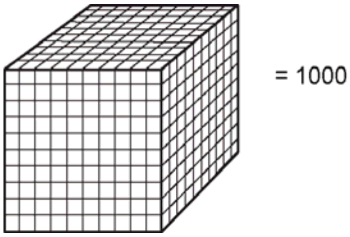
Correct Response A: Irrational numbers are numbers that cannot be written in fraction form, where both numerator and denominator are integers and the denominator is not zero. Pi (π) is one of the most well-known irrational numbers. In decimal form, representations of irrational numbers go on forever without a pattern. The sum of two irrational numbers is always irrational. One strategy to solve this is to eliminate the other three responses by finding a counter-example (one that gives a rational number for an answer).

Incorrect Response B: Any number subtracted from itself will equal zero, and zero is a rational number. It can be written as a fraction using integers for both numerator and denominator, for example, as $\frac{0}{7}$.

Incorrect Response C: Many examples can be given that result in rational products: for example, $\sqrt{5} \bullet \sqrt{5} = \sqrt{25} = 5$, which is a rational number because it can be written as a fraction using integers for both numerator and denominator (e.g., as $\frac{5}{1}$).

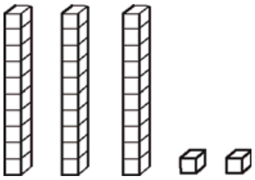
Incorrect Response D: A number divided by itself will always equal one, with the exception of zero. Division of zero by itself has no solution, but zero is not irrational, so it is not considered here. Any irrational number divided by itself will equal one. One is a rational number. It can be written in fraction form in many ways. For example, $\frac{3}{3}$ or $\frac{12}{12}$.

3. **Use the figure below to answer the question that follows.**

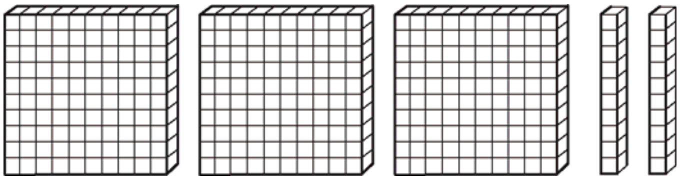


If the figure above represents 1000, which of the following figures represents 302?

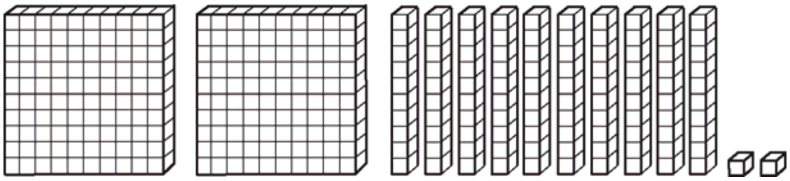
A.



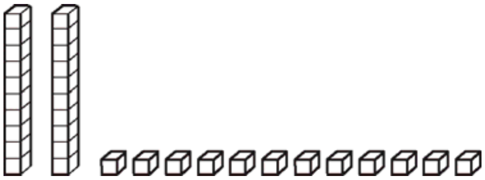
B.



C.



D.



Correct Response C: Analyzing the given information, each small cube represents one, each stack of ten cubes represents ten, each ten by ten layer of cubes represents 100, and the total figure given is ten layers of 100 cubes in each layer. The figure in response C represents 302. It is made up of 100 + 100 + 10 × 10 + 2 = 302.

Incorrect Response A: The figure here represents 10 + 10 + 10 + 2 = 32.

Incorrect Response B: The figure here represents 100 + 100 + 100 + 10 + 10 = 320.

Incorrect Response D: The figure here represents 10 + 10 + 12 = 32.

Each incorrect response shows a misinterpretation of the value for each arrangement of the cubes.

4. The base-10 number 827 is written in base 5 as 11302_5. What is the base-10 value of the digit 3 in this number?

A. 25

B. 75

C. 300

D. 375

Correct Response B: Base number systems all rely on place value or position for the value of the digit. No matter what the base is, each position will be a "power" of the base.

$\ldots\ldots\ldots b^5 \quad b^4 \quad b^3 \quad b^2 \quad b^1 \quad b^0$ where b equals the given base.

Remember that any number **except zero** raised to the zero power is equal to one.

Our number system is based on 10 so the values of each position are:

$\ldots\ldots\ldots 10^4 \quad 10^3 \quad 10^2 \quad 10^1 \quad 10^0$ or

$10\times10\times10\times10 \quad 10\times10\times10 \quad 10\times10 \quad 10 \quad 1$ or

10,000 1,000 100 10 1

The pattern is the same for base-5:

$\ldots\ldots\ldots 5^4 \quad 5^3 \quad 5^2 \quad 5^1 \quad 5^0$ or

$5\times5\times5\times5 \quad 5\times5\times5 \quad 5\times5 \quad 5 \quad 1$ or

625 125 25 5 1

The 3 in 11302_5 is in the position whose value is 5^2 so there are 3 groups of 25 and the base ten value of the 3 is 3×25 or 75.

Incorrect Response A: This response could have resulted from correctly identifying the place value as 5^2 but forgetting to multiply by 3.

Incorrect Response C: This response could have resulted from misinterpreting the question and evaluating the 3 as 3×10^2 in the base 10 system.

Incorrect Response D: This response could have resulted from adding the base ten and base five evaluations of the 3 in the b^2 positions.

5. For which of the following pairs of numbers is the number 3.45 larger than the first number, but smaller than the second number?

   A. $\frac{5}{2}$ and $\frac{13}{4}$

   B. $\frac{13}{4}$ and $\frac{7}{2}$

   C. $\frac{7}{2}$ and $\frac{15}{4}$

   D. $\frac{15}{4}$ and $\frac{9}{2}$

Correct Response B: One approach is to convert each of the improper fractions to its decimal form and then compare them: $\frac{13}{4} = 3\frac{1}{4} = 3.25$ —which is smaller than 3.45, and $\frac{7}{2} = 3\frac{1}{2} = 3.5 = 3.50$ — which is larger than 3.45.

An alternative approach is to convert each fraction to a fraction with the same denominator: $3.45 = 3\frac{45}{100} = 3\frac{9}{20} = \frac{69}{20}$. Each pair in the response can be rewritten with a denominator of 20: $\frac{13}{4} = \frac{65}{20}$ —this improper fraction is smaller than $\frac{69}{20}$; and $\frac{7}{2} = \frac{70}{20}$ —this improper fraction is larger than $\frac{69}{70}$.

Incorrect Response A: Compare in decimal form to 3.45, or in fraction form to $\frac{69}{20}$:

$\frac{5}{2} = 2.50$ and $\frac{13}{4} = 3.25$ —both are smaller than 3.45.
$\frac{5}{2} = \frac{50}{20}$ and $\frac{13}{4} = \frac{65}{20}$ —both are smaller than $\frac{69}{20}$.

Incorrect Response C: Compare in decimal form to 3.45 or in fraction form to $\frac{69}{20}$:

$\frac{7}{2} = 3.50$ and $\frac{15}{4} = 3\frac{3}{4} = 3.75$ —both are larger than 3.45.

$\frac{7}{2} = \frac{70}{20}$ and $\frac{15}{4} = \frac{75}{20}$ —both are larger than $\frac{69}{20}$.

Incorrect Response D: Compare in decimal form to 3.45 or in fraction form to $\frac{69}{20}$:
$\frac{15}{4} = 3.75$ and $\frac{9}{2} = 4\frac{1}{2} = 4.5$ —both are larger than 3.45.

$\frac{15}{4} = \frac{75}{20}$ and $\frac{9}{2} = \frac{90}{20}$ —both are larger than $\frac{69}{20}$.

6. The owner of a T-shirt shop bought a shipment of shirts, each for the same cost, and sold the shirts at 20% above cost. At the end of the season, the owner sold the remaining shirts at a sale: buy one and get the second shirt for half price. As a result, there was a loss on the sale of each of these shirts. The owner's loss per shirt was what percentage of the original cost?

   A. 10%

   B. 15%

   C. 20%

   D. 25%

Correct Response A: Let c = the owner's cost for one t-shirt. The owner adds 20% of the cost of one t-shirt as the mark-up. 20% is $\frac{20}{100} = \frac{2}{10} = 0.2$, so the mark-up will be 0.2c. Normally t-shirts will cost the buyer the owner's cost for the shirt plus the mark-up: 1c +0.2c = 1.2c.

Buy one, get one half price would be 1.2c for the first t-shirt and half of 1.2c or 0.6c for the other shirt: 1.2c + 0.6c = 1.8c for the two shirts. This means that the price of one of the t-shirts is 1.8c ÷ 2 = 0.9c. The owner bought the t-shirt for 1c but had to sell it at the end of the season for 0.9c. 1c - 0.9c = 0.1c. This is the loss on the t-shirt. The owner lost one-tenth of his cost on each of these shirts: 0.1 = .10 = 10% so there is a 10% loss on each t-shirt the owner had to sell this way.

Incorrect Response B: This response could have resulted from misinterpreting the conditions of the sale and using half the owner's cost (0.5c) instead of the marked-up price.

> 1.2c + 0.5c = 1.7c, 1.7c ÷ 2 = 0.85c, 1c - 0.85c = 0.15c or a 15% loss.

Incorrect Response C: This response could have resulted from using the owner's cost for the first shirt but correctly using half of the marked-up price for the second shirt.

> 1c + 0.6c = 1.6c for 2 shirts, 1.6c ÷ 2 = 0.8c for one shirt, 1c - 0.8c =0.2c or a 20% loss on each shirt.

Incorrect Response D: This response could have resulted from ignoring the 20% mark-up and using c, the owner's cost for the shirt, throughout the problem.

> 1c + 0.5c = 1.5c for 2 shirts, 1.5c ÷ 2 = 0.75c for one shirt, 1c – 0.75c = 0.25c or a 25% loss on each shirt.

7. Which of the following statements involving percentages is true?

   A. A person who has two credit cards paid 50% of the first credit-card bill of $648 and 45% of the second credit-card bill of $352. The person paid 95% of the total $1000 owed.

   B. A worker received an 8% raise at the end of the first year on the job and another 8% raise at the end of the second year. The worker's total raise was 16% of the starting salary.

   C. In a particular county, 15% of the population is over 65 years old, 40% is 35 to 65 years old, 30% is 18 to 34 years old, and 20% is under 18 years old.

   D. A town spent 36% of its annual budget on public safety and 10% on public works projects. The town spent 46% of its annual budget on public safety and public works projects.

Correct Response D: The percents are each referring to the same budget and so the same amount of money. This can be shown algebraically.

Let B = the amount of money in the town's budget:

$$36\,\% = \frac{36}{100} = 0.36 \text{ and } 10\,\% = \frac{10}{100} = 0.10$$

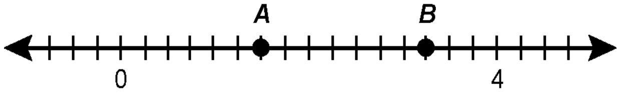
36% of the annual budget = $0.36B$ and 10% of the annual budget = $0.10B$, then, $0.36B + 0.10B = 0.46B$, which is 46% of the total budget.

Incorrect Response A: The percents in this situation are each taken from different amounts of money. Algebraically, that is adding 0.5x + 0.45y. The variables are unlike terms and cannot be added. 50% of $648 means that the person paid half of the bill, or $324. 45% of the second bill would be .45 × 352, or $158.40. The person paid $482.40 out of the $1000 owed. This is less than half of the $1000, so it represents a percent that is less than 50%.

Incorrect Response B: The percents cannot be combined for the same reason as given for response A. If s = worker's salary and 8% is $\frac{8}{100} = 0.08$, then the raise may be represented by $0.08s$. The worker is now earning the old salary plus the raise, $s + 0.08s = 1.08s$, for the second year. The second year raise will be on the salary the worker earned that year, $1.08s$. The second year raise will be $0.08(1.08s) = 0.0864s$. The total raise over the two years will be $0.08s$ from year one and $0.0864s$ from year two for a total of 0.1664 of the original salary, or 16.64% of the original salary in raises.

Incorrect Response C: Here, all of the percents are referring to the same total population, so they may be added, but when these percents are added the result is larger than 100%: 15% of the population + 40% of that population + 30% of that population + 20% of that population = 105% of the population. The **whole** population is represented by 100%. Since the groups represented have no members in common, the total cannot exceed 100% of the population.

8. **Use the number line below to answer the question that follows.**



Which of the following numbers is located between point *A* and point *B* on the number line above?

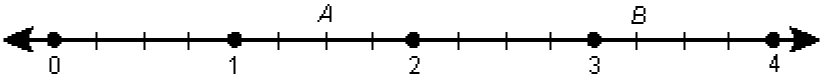
A. $\frac{3}{8}$

B. $\frac{9}{16}$

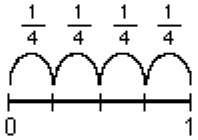
C. $2\frac{1}{4}$

D. $3\frac{1}{2}$

Correct Response C: To read this number line, the size of the intervals (distance between the tick marks) needs to be determined. Zero and four are given with 16 intervals 16 ├--------┤ lengths from zero to four. This means 16 intervals ÷ 4 wholes = 4 intervals in each whole. Four intervals in one whole means that the whole is cut into four equal parts, cut into fourths.



The distance between tick marks is what is counted. Four of these lengths equal 1.



Point *A* is located at $1\frac{2}{4} = 1\frac{1}{2}$. Point *B* is located at $3\frac{1}{4}$. A number between these two amounts is needed. $2\frac{1}{4}$ is the only choice that is more than $1\frac{1}{2}$ but less than $3\frac{1}{4}$. It is located one interval to the right of 2, which places it between point *A* and point *B*.

Incorrect Response A: This response might have come from counting 16 spaces between 0 and 4, but then counting each space as $\frac{1}{16}$. This means that point *A* would be erroneously labeled $\frac{6}{16}$ and point *B* would be erroneously labeled $\frac{13}{16}$. $\frac{3}{8} = \frac{6}{16}$, which would be located at point *A*.

Incorrect Response B: This response might also have come from counting 16 spaces between 0 and 4. $\frac{9}{16}$ is a number located between *A*, which would be erroneously labeled $\frac{6}{16}$, and *B*, which would be erroneously labeled $\frac{13}{16}$.

Incorrect Response D: In this response, the size of the intervals is correctly determined but the number of intervals is incorrectly counted. $3\frac{1}{2} = 3\frac{2}{4}$, which is located one tick mark to the right of point *B*.

9. **Use the problem below to answer the question that follows.**

> After driving 180 miles, a family had completed $\frac{5}{8}$ of their trip. How many more miles must they drive to complete their entire trip?

Which of the following expressions models the solution to the problem above?

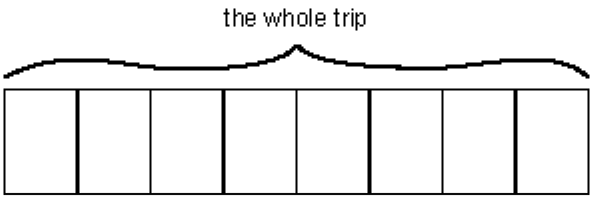
A. $\frac{3}{8}(180)$

B. $\frac{3}{5}(180)$

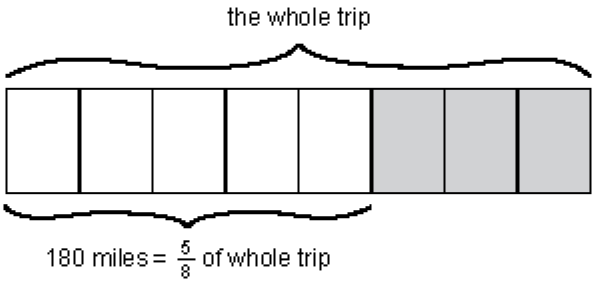
C. $\frac{8}{3}(180)$

D. $\frac{5}{3}(180)$

Correct Response B: There are several ways to arrive at this solution. One way is to use a diagram. Let the rectangle represent the whole trip. Since the information given is in eighths, divide the rectangle into 8 equal parts.



The problem states that after 180 miles, five-eighths of the trip is completed. This means that 180 miles must be equal to five-eighths of the trip and therefore five-eighths of the rectangle.



Notice that all of the response choices relate to the 180. The diagram shows that five equal parts of the trip are in the 180 miles, so each of the unshaded blocks is one-fifth of 180. The rest of the trip is three of these equal sections, represented by the three shaded blocks, and is the same as $\frac{3}{5}$ of 180, which is found by multiplying $\frac{3}{5} \times 180 = \frac{3}{5}(180)$. This is the same as cutting the 180 into five equal parts to find out how many miles is in one-fifth and then multiplying by 3 to find out how many miles is in three-fifths.

Another way to solve this problem is to use proportions: If $\frac{5}{8}$ of the trip is completed, then $\frac{3}{8}$ of the trip is left. $\frac{180 \ miles}{\frac{5}{8} \ of \ \ trip} = \frac{x \ \ miles}{\frac{3}{8} \ of \ \ trip}$, then cross multiply to get $\frac{3}{8}(180) = \frac{5}{8}x$. Then multiply each side by the reciprocal of $\frac{5}{8}$, which is $\frac{8}{5}$, to get $\frac{8}{5} \bullet \frac{3}{8}(180) = \frac{8}{5} \bullet \frac{5}{8}x$. Now notice that $\frac{8}{5} \bullet \frac{3}{8} = \frac{24}{40} = \frac{3}{5}$, and $\frac{8}{5} \bullet \frac{5}{8} = 1$. This simplifies the equation to $\frac{3}{5}(180) = x$.

Incorrect Response A: This response correctly identifies that $\frac{3}{8}$ of the whole trip is left. The distance of the whole trip is unknown, so the distance left to travel is $\frac{3}{8}$ of an unknown amount, not $\frac{3}{8}$ of the 180 miles already traveled.

Incorrect Response C: This response correctly identifies that $\frac{3}{8}$ of the whole trip is left, but uses an incorrect operation on the 180 miles already traveled: $180 \div \frac{3}{8} = 180 \bullet \frac{8}{3}$.

Incorrect Response D: This response may have used the proportion method given in the explanation for response B, but divided by $\frac{3}{5}$ instead of multiplying. $180 \div \frac{3}{5} = 180 \bullet \frac{5}{3}$. Note that if the diagram is used, it is clear that 180 should be divided by 5, not 3.

10. In which of the following lists are the numbers correctly ordered from least to greatest?

    A. 8%, $\frac{2}{5}$, 0.16, $\frac{3}{8}$

    B. $\frac{1}{8}$, 2.5%, $\frac{1}{4}$, 0.05

    C. 0.9, $\frac{1}{2}$, 75%, $\frac{3}{5}$

    D. 130%, $\frac{8}{5}$, 2.08, $\frac{10}{3}$

Correct Response D: To compare the numbers more easily, it would be helpful to convert each to its decimal equivalent. A comparison could also be done, changing each of them to the equivalent fraction and then finding the common denominator.

130% is the same as $\frac{130}{100} = 1\frac{30}{100} = 1.30$, $\frac{8}{5} = 1\frac{3}{5} = 1\frac{6}{10} = 1.6$, 2.08 is already in decimal form, and $\frac{10}{3} = 3\frac{1}{3} = 3.\overline{3}$ (1 ÷ 3 gives .3 with the 3 repeating forever, which is the meaning of the bar above the decimal 3). Now that all are in a decimal form, they can be arranged from lowest to highest: 1.3, 1.6, 2.08, $3.\overline{3}$, or in the original forms, 130%, $\frac{8}{5}$, 2.08, $\frac{10}{3}$.

Incorrect Response A: When each is converted to decimal form:

$8\% = \frac{8}{100} = 0.08$, $\frac{2}{5} = \frac{4}{10} = 0.4$, 0.16 is already in decimal form, and $\frac{3}{8} = \frac{375}{1000} = 0.375$ (multiply both the numerator and denominator by 125 or simply divide 3 by 8 to get .375).

Note that this response can be eliminated once 0.16 is compared to 0.4, since 0.4 = 0.40 and is larger than 0.16.

Incorrect Response B: When each is converted to decimal form:

$\frac{1}{8} = 0.125$, $2.5\% = \frac{2.5}{100} = \frac{25}{1000} = 0.025$, $\frac{1}{4} = 0.25$, and 0.05 is already in decimal form.

As in response A, work could have stopped after comparing $\frac{1}{8} = 0.125$ and 2.5% = 0.025 since 0.025, is smaller than 0.125.

Incorrect Response C: When each is converted to decimal form:

0.9 = 0.90, $\frac{1}{2} = 0.5 = 0.50$, 75% = 0.75, and $\frac{3}{5} = 0.6 = 0.60$.

As in the last two responses, work could stop after comparing 0.9 and 0.5, since 0.5 is smaller than 0.9.

11. If $a = 2^3 \cdot 3 \cdot 5$ and $b = 2^2 \cdot 3^2 \cdot 7$, what is the result of dividing the least common multiple of *a* and *b* by the greatest common factor of *a* and *b*?

    A. 70

    B. 140

    C. 210

    D. 420

Correct Response C: The quotient of the LCM (lowest common multiple) of *a* and *b* divided by the GCF (greatest common factor) is 210.

Notice that *a* and *b* have been written as their prime factorizations. This will make finding the LCM and GCF easier. The lowest common multiple of two numbers is the smallest product containing both of them. $a = 2^3 \cdot 3 \cdot 5$, so that will have to be part of the multiple. The second number, $b = 2^2 \cdot 3^2 \cdot 7$, contains some of the same factors as *a*.

To start to find the LCM, $a = 2^3 \cdot 3 \cdot 5$ is needed. The LCM must be a multiple of *a*. The factors in *b* that are not already in *a* must be included so that the result is a multiple of *b* also. As few factors as possible are used so that it will become the lowest multiple that the two numbers share in common. 2^3 contains $2 \cdot 2^2$, so that factor of *b* is not needed. It is there already.

$2^3 \cdot 3 \cdot 5$ contains one factor of 3 but *b* has 3^2 or $3 \cdot 3$, so one more factor of 3 is needed in the LCM. There is no factor of 7 in the first number, so that will also be needed. The result is: LCM = $2^3 \cdot 3^2 \cdot 5 \cdot 7$. This is the smallest set of factors multiplied together that contain both *a* and *b*. It is the smallest product that has both *a* and *b* as factors. To test this, look at the LCM and find *a* (three 2's, two 3's, and a 5) then find *b* (two 2's, two 3's and a 7).

The greatest common factor of *a* and *b* will be the factors that they share in common. An easy way to find them is to write out each prime factorization without using exponents.

$a = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$ and $b = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$. The only factors that they share in common are $2 \cdot 2 \cdot 3$. The product of this is the GCF.

LCM ÷ GCF will be $\frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 2 \cdot 3}$. Common factors in the top and bottom may be reduced to 1: $\frac{2}{2} = 1$ and $\frac{3}{3} = 1$. The result is $2 \cdot 3 \cdot 5 \cdot 7 = 210$.

Another approach to the division would be to multiply out the LCM and the GCF.

LCM = $2^3 \cdot 3^2 \cdot 5 \cdot 7 = 2520$ and the GCF = $2 \cdot 2 \cdot 3 = 12$, and $2520 \div 12 = 210$.

Incorrect Response A: This response uses the prime factor with the lowest exponent in each, getting $2^2 \cdot 3 \cdot 5 \cdot 7$, which is not a multiple of either *a* or *b*, then dividing by the two prime numbers that they share in common, the 2 and the 3. Although they are common factors, their product is not the GCF.

Incorrect Response B: This response multiplies *a* and *b* together to get $2^5 \cdot 3^3 \cdot 5 \cdot 7$ and uses the expression $2^3 \cdot 3^3$ as the GCF. $2^5 \cdot 3^3 \cdot 5 \cdot 7$ is a multiple of both *a* and *b* but not the lowest multiple common to both, and the GCF is incorrect because it has an extra factor of 3.

Incorrect Response D: This response multiplies *a* and *b* together as response B did, but then uses the largest exponent on the common factors instead of the shared factors.

12. If *m* and *n* are both prime numbers greater than two, then the number represented by (*mn* + 1) must be:

    A. even.

    B. odd.

    C. prime.

    D. a perfect square.

Correct Response A: If *m* and *n* are prime numbers larger than 2, it means that they must each be an odd number. Two is the only even prime number because any even number larger than 2 will have 2 as a factor. An odd number multiplied by an odd number will always result in a product that is an odd number. When one is added to an odd number, the result is an even number.

To eliminate the other results, look for a counter-example in each case, an example where the answer is not true and therefore proves the statement false.

Incorrect Response B: Let *m* = 3 and *n* = 5. Then 3 • 5 + 1 = 16, which is an even number, not an odd number.

Incorrect Response C: Let *m* = 3 and *n* = 5. Then 3 • 5 + 1 = 16, which is not a prime number. A prime number has only one and itself as factors.

Incorrect Response D: Let *m* = 3 and *n* = 7. Then 3 • 7 + 1 = 22, which is not a perfect square. Notice that if we had used *m* = 3 and *n* = 5, the result is 16, which is a perfect square (4 • 4). If this happens, it is important to try another pair of prime numbers. Only one case where it is not true makes the statement false.

13. When the number 3600 is written in the form $x^a y^b z^c$, where x, y, and z are prime numbers, what is the value of $a + b + c$?

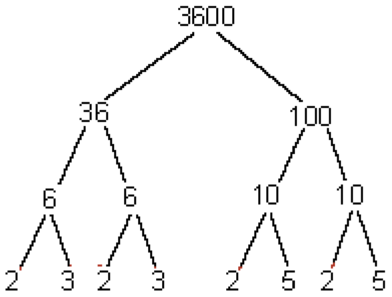
    A. 5

    B. 6

    C. 8

    D. 10

Correct Response C: A prime factorization represents a number as the product of all of its prime factors. An easy way to find the prime factorization of a number is to use a factor tree. The factor tree will look different depending on the factors used but the bottom of the tree should always contain only prime numbers and therefore look the same no matter what original factors were used.



Notice that all the factors in the bottom row of the factor tree are prime numbers. The process of finding factors needs to continue until all of the factors in a row are prime numbers. This is the prime factorization of the original number.

$3600 = 2 \cdot 3 \cdot 2 \cdot 3 \cdot 2 \cdot 2 \cdot 5 \cdot 5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 = 2^4 \cdot 3^2 \cdot 5^2$, so the prime factorization of 3600 is $2^4 \cdot 3^2 \cdot 5^2$. Comparing this to $x^a y^b z^c$, $a = 4$, $b = 2$ and $c = 2$. The sum $a + b + c = 4 + 2 + 2 = 8$.

Incorrect Response A: This response might have come from an incomplete factorization, using $4^2 \cdot 5^2 \cdot 9^1$, where 4 and 9 are not prime. $2 + 2 + 1 = 5$, which is the sum of the exponents.

Incorrect Response B: This response might be a misinterpretation of what was asked for, making all the exponents to be the same. $4^2 \cdot 3^2 \cdot 5^2 = 3600$, but 4 is not prime. $a + b + c = 2 + 2 + 2 = 6$.

Incorrect Response D: This response might be a misinterpretation of how x, y, and z were defined. The three bases are added rather than the three exponents: $2 + 3 + 5 = 10$.

14. Which of the following statements is enough to confirm that a number *N* is divisible by 36?

    A. *N* is divisible by 6.

    B. *N* is divisible by both 4 and 9.

    C. *N* is divisible by both 3 and 12.

    D. *N* is divisible by both 2 and 18.

Correct Response B: The numbers 4 and 9 are relatively prime. This means that they share no common factors. If a number is divisible by 4 (the last two digits can be divided by four with no remainder) and divisible by 9 (the sum of the digits in the number are divisible by 9) then it must also be divisible by 36, the product of the two numbers. Note that 36 = 4 • 9 = 2 • 2 • 3 • 3.

Each of the other responses will not necessarily work because they do not take into consideration the factors that are relatively prime.

Incorrect Response A: All multiples of 6 can be divided by 6 but not all of these can be divided by 36. For example: 6, 12, 18, 24, 30, 36, 42, 48, 54, and 60 are the first ten non-zero multiples of 6, which all are divisible by 6 but only one of which, 36, is also divisible by 36.

Incorrect Response C: The first six non-zero multiples of 12 are 12, 24, 36, 48, 60, and 72. Any multiple of 12 is also divisible by 3 since 3 • 4 is 12. Of the six multiples listed, only two are divisible by 36, though all are divisible by 3 and 12.

Incorrect Response D: The first seven non-zero multiples of 18 are 18, 36, 54, 72, 90, 108, and 126. Any multiple of 18 is also divisible by 2 since 2 • 9 is 18. The only ones divisible by 36 are 36, 72, and 108, even though all seven are divisible by 2 and 18.

15. Which of the following problems can be solved by calculating the least common multiple of the given quantities?

    A. A surveyor needs to divide a rectangular plot of land 840 meters wide and 1680 meters long into equally-sized square plots. If the dimensions of the plots need to be whole numbers, what is the size of the largest plots the surveyor can create?

    B. The environmental action club at a school has 13 girls and 15 boys. Five club members will be chosen to attend a workshop. In how many ways can the five students be chosen if two of them must be girls?

    C. A jar contains 8 blue marbles, 12 red marbles, and 15 green marbles. If three marbles are drawn together from the jar, what is the probability that each of the three marbles drawn is a different color?

    D. In a party supply shop, paper plates are sold in packages of 30, cups in packages of 24, and napkins in packages of 36. What is the smallest number of plates, cups, and napkins that can be bought so that there is an equal number of each?

Correct Response D: It can be solved by using the LCM (Lowest Common Multiple). The smallest multiple of 24, 30, and 36 must be found. Factor trees can be used to find the prime factorization of each of these numbers (see factor tree explanation in the rationale text for problem 14).

$24 = 2^3 \cdot 3$ $30 = 2 \cdot 3 \cdot 5$ $36 = 2^2 \cdot 3^2$

To find the LCM, the new number must contain each of the original numbers or else the new amount would not be a multiple of it. Look at the prime factorization of 24: The LCM must contain 2^3 and 3. Now look at the prime factorization of 30: For the LCM to be a multiple of 30 it must contain a 2, a 3, and a 5. In the prime factorization of 24 there is already a 2 and a 3 but there is no 5. A 5 is needed. The LCM of 24 and 30 is $2^3 \cdot 3 \cdot 5 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$. The product of the first four factors = 24 and the product of the last three factors = 30. To make this product also be a multiple of 36, two factors of 2 and two factors of 3 are required. ($36 = 2^2 \cdot 3^2$). $2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$ already contains the needed 2^2 and one of the factors of 3, therefore one more factor of 3 must be included in the LCM.

The LCM of 24, 30, and 36 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 3$ or $2^3 \cdot 3^2 \cdot 5 = 360$. $15 \cdot 24 = 360$ and $12 \cdot 30 = 360$ and $10 \cdot 36 = 360$. In order to have 360 of each of the three items, 15 packages of cups, 12 packages of paper plates, and 10 packages of napkins would be needed.

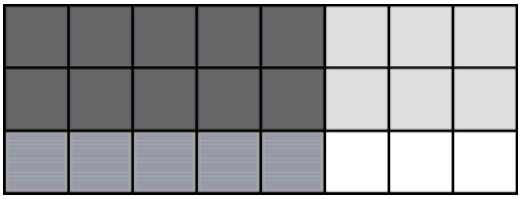
An alternative way to approach the solution would be to find the prime factorization of each of the numbers but start with the prime factors rather than with the individual numbers, 24, 30, and 36. Find the largest power of two in the numbers given. 2^3 is needed for the multiple of 24. 2^3 contains 2^1, needed for the 30, and 2^2, needed for the 36. Find the largest power of 3 needed to make the numbers given. 3^2 is needed to make a multiple of 36. 3^2 contains 3^1, or 3, needed for the 30 and 24. Now the only missing factor is the 5 needed for 30 and the LCM is $2^3 \cdot 3^2 \cdot 5$.

Incorrect Response A: This problem needs the GCF (greatest common factor) of 840 and 1680 to solve it. The largest number that can divide into both of them will lead to the solution.

Incorrect Response B: This problem requires counting the number of ways to choose 5 students from the groups given. It uses the theory of combinations.

Incorrect Response C: This is a probability problem that requires finding all combinations of drawing three marbles at once, seeing how many of those combinations result in one of each color marble, and comparing that number to the total number of combinations of three marbles of any colors.

16. **Use the diagram below to answer the question that follows.**



Which of the following problems can be solved using the operation model shown?

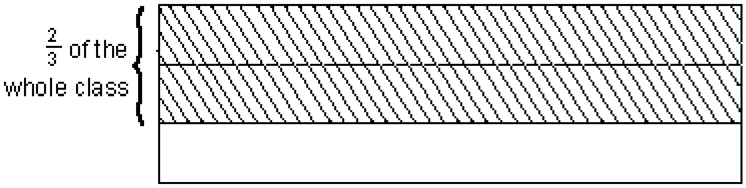
A. In a bag of marbles, $\frac{5}{8}$ of the marbles are red. In a second bag, $\frac{2}{3}$ of the marbles are blue. If the two bags are combined into one bag, what is the probability of picking a red marble followed by a blue marble, if the red marble is not replaced?

B. The results of a survey in a third-grade class showed that $\frac{2}{3}$ of the students had at least one pet and that $\frac{5}{8}$ of the students who had pets had at least one dog. What fraction of the class had at least one dog?

C. There are two pints of milk. A child drinks $\frac{5}{8}$ of one pint and the child's friend drinks $\frac{2}{3}$ of the second pint. What fraction of a pint remains?

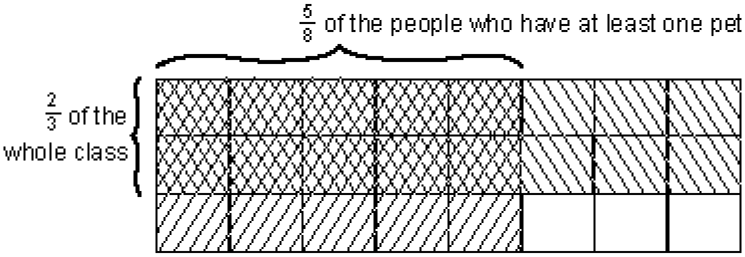
D. One person can do a particular job in $\frac{2}{3}$ of an hour. A second person can do the same job in $\frac{5}{8}$ of an hour. What is the ratio of the first person's time to the second person's time?

Correct Response B: The problem states that $\frac{2}{3}$ of the whole third-grade class has at least one pet. This can be diagrammed as:



The whole rectangle represents the third-grade class. It is cut into thirds and two of those sections are shaded. The shaded part represents the students in the class that have at least one pet.

The problem states that $\frac{5}{8}$ of the children who have at least one pet have a dog. This is now referring to the shaded part of the diagram only. Five-eighths of the shaded part are those children who have at least a dog. The shaded part needs to be cut into eight equal sections, with five of those sections shaded.



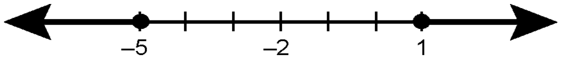
The double-shaded area represents $\frac{5}{8}$ of $\frac{2}{3}$ of the class.

Incorrect Response A: This situation puts all the marbles into a third bag, changing the total (the whole), and the fractions given do not represent the new whole. For example, if each bag contained 24 marbles there would be 15 red in the first bag ($\frac{5}{8}$ of 24) and 8 in the second bag ($\frac{1}{3}$ of 24). Combined, 23 out of 48 marbles are now red, which is a totally different fraction than either of the two given.

Incorrect Response C: This solution requires that $\frac{2}{3}$ and $\frac{5}{8}$ be added to see how much has been drunk, and then that total needs to be subtracted from one pint. Another approach would be to find how much of each of the pints is left using subtraction, and then add the results together to find the total.

Incorrect Response D: The solution to this problem would involve finding the ratio of $\frac{2}{3}$ to $\frac{5}{8}$, or $\frac{\frac{2}{3}}{\frac{5}{8}} = \frac{2}{3} \div \frac{5}{8} = \frac{2}{3} \times \frac{8}{5}$, or $\frac{2}{3}$ of more than one whole, which is not the operation model given.

17. **Use the number line below to answer the question that follows.**



Which of the following inequalities best represents the graph shown on the number line?

A. $| x – 2 | \geq 3$

B. $| x + 2 | \geq 3$

C. $| x – 3 | \geq –2$

D. $| x + 3 | \geq –2$

Correct Response B: The symbol | | is called absolute value. The absolute value of a number is its distance from zero. $|4| = 4$ and $|–4| = 4$ because both 4 and –4 are 4 units from zero. When distances between numbers are calculated, subtraction is used; to get the distance between 25 and 60, subtract 25 from 60, which is 35. If the subtraction is done "backward," i.e., 25 – 60, a negative number results. Distance is not negative, so using absolute value solves this problem: $|25 – 60| = |–35| = 35$. Now, no matter which order the numbers are subtracted in, the distance is 35. Looking at the number line, notice that the darkly shaded rays are placed so that the graph has symmetry around –2. The numbers in the darkened ray on the right are 3 or more units away from the –2. The numbers in the darkened ray on the left are also 3 or more units away from the –2. Remember that distance is found by subtraction, so to write "the distance between some number called x and the number –2," use $|x – (–2)| = |x + 2|$. Now, the distances between the x numbers and –2 are 3 or more, so $|x + 2| \geq 3$.

Incorrect Response A: This response could have come from incorrectly expressing the distance between x and –2 as $|x – 2|$ rather than $|x – (–2)|$.

Incorrect Response C: This response could have come from confusing the positions of the –2 and the 3 in the equation when trying to express distance from –2. $|x – 3|$ is the expression for the distance between x and 3.

Incorrect Response D: This response could have come from confusing the positions of the –2 and 3 in the distance equation and then incorrectly writing distance as addition rather than subtraction.

18. What is the value of the expression $\sqrt{9+16} - 12 \div 2^2 + 3 \times (9 - 5)$?

    A. 5

    B. 7

    C. 14

    D. 16

Correct Response C: The value of the expression is 14. The properties of different operations in mathematics require them to be done in a particular order. This is called "order of operations." For example: 7 + 3 • 4 means 7 added to 3 groups of 4.

The "7" represents an amount of things, a quantity, but the "3" represents groups of those things. A simple word problem to illustrate this would be: Four children picked apples at an orchard. Ann picked 7 apples. Each of her three friends picked four apples. How many apples did they pick altogether?

The 7 represents the amount of apples, as does the 4, but the 3 represents 3 *people*. Apples and people cannot be added since they are different. The multiplication must be done before the addition. Three people each have 4 apples, so 4 + 4 + 4 or 3 groups of 4 or 3 • 4 = 12 apples. Seven apples from Ann plus 12 apples from the friends equal 19 apples.

The order of operations states parentheses and exponents are done first, then multiplication and division are done in the order that they appear reading from left to right. After these operations are done, addition and subtraction are done in the order that they are present from left to right.

In the given expression, the $\sqrt{9+16}$ and (9–5) should be simplified first. Square roots and other roots are like having parentheses. $\sqrt{9+16} = \sqrt{25} = 5$ and (9 – 5) = 4, so the new expression is $5 - 12 \div 2^2 + 3 \cdot 4$. The 2^2 is done next: $2^2 = 2 \cdot 2 = 4$. Now the expression is $5 - 12 \div 4 + 3 \cdot 4$.

Subtraction, division, and multiplication remain to be done. By the order of operations, multiplication and division are done next from left to right. In this case, division of 12 by 4 comes first, then 3 is multiplied by 4, and 5 – 3 + 12 is what is left of the expression. Lastly, addition and subtraction are done in order from left to right: 5 – 3 = 2 then 2 + 12 is 14.

Incorrect Response A: This response results from performing the operations in the order written from left to right, ignoring the order of operations rules.

Incorrect Response B: In this case, the $\sqrt{9+16}$ is computed as $\sqrt{9} + \sqrt{16}$, which is not true. The square root of the sum of 9 and 16 is the square root of 25 or 5, but the square root of 9 is 3 and the square root of 16 is 4, giving 3 + 4 = 7 as the result. Also, the rest of the computation was done from left to right, again ignoring the order of operations.

Incorrect Response D: This response uses the same square root error as in response B, but applies order of operations rules and does the rest of the evaluation correctly.

19. What is the value of the expression $\left(3\frac{1}{4} - 1.019\right) \div 0.07$ rounded to the nearest tenth?

A. 31.9

B. 32.1

C. 34.1

D. 34.6

Correct Response A: In order of operations, calculations in parentheses are performed before division. (Refer to the previous question to review order of operations.) The mixed number $3\frac{1}{4} = 3.25$ or 3.250 (the zero was not necessary, but helps in the subtraction).

3.350 2.231 ÷ 0.07 = 31.87 or $.07\overline{)2.231}$,

moving the decimal point to get $\begin{array}{r} 31.87 \\ 7\overline{)223.10} \end{array}$

$\begin{array}{r} -\underline{1.019} \\ 2.231 \end{array}$

The quotient is 31.87 to two decimal places. The digit in the hundredths place is 5 or larger, so the tenths digit is increased by one. 31.87 to the nearest tenth is 31.9.

Incorrect Response B: In this response, the subtraction is done incorrectly by "bringing down" the 9 and not subtracting it from the 3.350, resulting in 2.249 for the subtraction. The division is then done correctly using the incorrect difference.

Incorrect Response C: In this response, $3\frac{1}{4}$ is converted to 3.4, then the rest of the computation proceeds correctly from this error.

Incorrect Response D: In this response, $3\frac{1}{4}$ is again converted to 3.4, the subtraction "brings down" the 1 and 9, not subtracting those digits from 3.400; this results in 2.419, which is incorrect. The division was done correctly using the 2.419.

20. **Use the calculation below to answer the question that follows.**

To multiply 13 × 45:

1 × 45 = 45
2 × 45 = 90
4 × 45 = 180
8 × 45 = 360

Since 13 = 8 + 4 + 1, then 13 × 45 = 360 + 180 + 45 = 585

The strategy above is based on which of the following properties of real numbers?

A. identity

B. commutative

C. reflexive

D. distributive

Correct Response D: The distributive property is often written as $a (b + c) = a \bullet b + a \bullet c$, but it also applies to adding more than 2 numbers. It can be extended to $a (b + c + d + e + \dots) = a \bullet b + a \bullet c + a \bullet d + a \bullet e + \dots\dots$ Due to the commutative property of multiplication, this expression can also be written in reverse order: $(b + c + d) a = b \bullet a + c \bullet d + d \bullet a$.

This strategy is using multiplication with the addition of three numbers: 13 • 45 = (8 + 4 + 1) • 45, which is equal to 8 • 45 + 4 • 45 + 1 • 45.

The 13 groups of 45 are being written as 8 groups of 45 + 4 groups of 45 + 1 group of 45, using the distributive property.

Incorrect Response A: The identity properties say that any number + 0 = the number and any number × 1 = the number. Neither of the identity properties is used here.

Incorrect Response B: The commutative property states that two numbers can be added or multiplied in any order and the result will be the same: $a \bullet b = b \bullet a$, or $a + b = b + a$. The strategy shown uses multiplication combined with addition. The commutative property deals with only one operation at a time.

Incorrect Response C: The reflexive property says that a number is equal to itself. The reflexive property is not in use here.

21. A home products store sells two types of grass seed.

- A 50-pound bag of bluegrass costs $165.
- A 50-pound bag of fescue costs $108.

In a particular week, the store sold 8 more bags of fescue than of bluegrass. The income from the bags of bluegrass was $294 less than the income from the bags of fescue. Which of the following equations could be solved to find b, the number of bags of bluegrass sold that week?

A. $108(b + 8) = 165b + 294$

B. $108b + 294 = 165(b - 8)$

C. $108(b + 8) + 294 = 165b$

D. $108b = 165(b - 8) + 294$

Correct Response A: To set up an equation to solve for b, look for given information that allows for relationships to be set equal to each other. Two pieces of information are given here: the relationship between the number of bags sold and the relationship between the incomes earned from each type of grass.

It was given that the variable, *b*, represents the number of bags of bluegrass sold. The problem states that 8 more bags of fescue were sold than of bluegrass. This (the number of bags of fescue sold) would be shown by adding 8 to the number of bags of bluegrass, i.e., *b* + 8.

The other information given states that the income earned by selling bags of bluegrass was $294 less than was earned selling bags of fescue. To make an equation, an equality is needed. Both sides of the equation must represent the same amount (in this case, the same amount of money).

Each 50 lb. bag of bluegrass costs $165. This means that 165*b* represents the amount of money earned selling these bags of seed (for example, if you had 6 bags of bluegrass, 165 would be added six times, or 6 times 165).

Each 50 lb. bag of fescue seed costs $108 and there are *b*+8 of them. (*b*+8) times 109 will equal the income from the bags of fescue seed. This is written as 108(*b*+8). Note that the *b*+8 was put into parentheses. The total of *b*+8 is the number of bags, and all of it must be multiplied by the price of 108.

The information given states that 294 fewer dollars were earned selling the bluegrass bags. 165*b* represents a number that is 294 less than the number that 108(*b*+8) represents. If 294 is added to the smaller amount (165*b*), then each expression would represent the same amount:
108(*b*+8) = 165b + 294.

In words, the equation essentially states: "the money earned selling bags of fescue is equal to the money earned when $294 is added to the money made selling bags of bluegrass seed."

Incorrect Response C: 108(*b*+8) is already larger than 165*b* by the information given. The information states that it is $294 more. Adding another $294 to it will yield a number 2(294) more than 165*b*. For example, if you wish to make an equality using 8 and 20, then 12 must be added to the smaller number, 8, so that 8 + 12 = 20. The 294 must be added to the smaller amount of money, 165*b*, in order to make each side of the equation represent the same number.

Incorrect Response B: Two errors are made here. One error is the same as found in response A, adding 294 to the already larger number. The second error is representing the number of bags of bluegrass as *b* – 8. Eight *more* bags of fescue seed were sold, but *b* – 8 represents 8 fewer bags of fescue sold.

Incorrect Response D: This error is discussed in response B. The 294 is added to the correct side of the equation but, again, *b* + 8 is needed, not *b* – 8.

22. The formula $L = \pi(r_1 + r_2) + 2d$ calculates the length L of a belt around two pulleys whose radii are r_1 and r_2 if the distance between their centers is d. Which of the following formulas could be used to calculate r_1, the radius of one of the pulleys?

A. $r_1 = \pi(L - 2d) - r_2$

B. $r_1 = \dfrac{L - 2d}{\pi} - r_2$

C. $r_1 = \dfrac{L - 2d - r_2}{\pi}$

D. $r_1 = \dfrac{L - 2d}{\pi r_2}$

Correct Response B: The formula to calculate r_1 is obtained by "solving the given equation for r_1." This means the r_1 must be isolated (get r_1 = all the other terms) by using algebraic steps: $L = \pi(r_1 + r_2) + 2d$. Think of the steps in this formula that must be done to find L. First, add r_1 and r_2 because calculations in parentheses are solved first in the order of operations. Second, multiply that sum by π, and then lastly add $2d$. To isolate r_1, this all must be "undone" by working backwards. The last step done was to add $2d$, therefore $2d$ must be subtracted. Next, the sum of $r_1 + r_2$ was multiplied by π, so the reverse must be done, which is to divide by π. Lastly, r_2 was added to r_1, so it needs to be subtracted.

$2d$ can be subtracted from both sides of the equality: $L - 2d = \pi(r_1+r_2) + 2d - 2d$, which simplifies to $L - 2d = \pi(r_1+r_2)$,

Each side can be divided by π (to "undo" multiplying (r_1+ r_2) by π): $\dfrac{L - 2d}{\pi} = \dfrac{\pi(r_1 + r_2)}{\pi}$, which simplifies to $r_1 + r_2 = \dfrac{L - 2d}{\pi}$.

Lastly, r_2 needs to be subtracted from each side of the equation so that r_1 is by itself: $r_1 + r_2 - r_2 = \dfrac{L - 2d}{\pi} - r_2$. Simplified, this becomes $r_1 = \dfrac{L - 2d}{\pi} - r_1$.

Incorrect Response A: This response might come from multiplying each side of the equation by π when in fact it is division by π that is needed.

Incorrect Response C: This response may result from trying to subtract r_2 before dividing by π.

Incorrect Response D: This response distributes $\pi(r_1 + r_2)$ to get $\pi r_1 + \pi r_2$, which is a correct strategy. Each side is divided by πr_2 when subtraction of πr_2 is needed, and the π that was part of πr_1 was lost.

23. The ratio of adults to children on a bus was 4 to 1. When 15 adults got off at the next stop and no one else got on, the ratio of adults to children became 5 to 2. Which of the following equations could be used to find c, the number of children on the bus?

A. $\dfrac{4c - 15}{c} = \dfrac{5}{2}$

B. $\dfrac{\frac{1}{4}c - 15}{c} = \dfrac{5}{2}$

C. $\dfrac{c}{4c - 15} = \dfrac{5}{2}$

D. $\dfrac{c}{\frac{1}{4}c - 15} = \dfrac{5}{2}$

Correct Response A: To create a valid equation, two ratios are needed that relate the number of adults to children on the bus after the bus stop. One ratio given is 5 adults for every 2 children after 15 adults got off, so $\dfrac{\#\,adults}{\#\,children} = \dfrac{5}{2}$. Another ratio given was 4 adults for each child on the bus before 15 adults got off the bus. This ratio means that there were four times as many adults on the bus as there were children. If c is defined to be the number of children on the bus before the stop, then 4 times c (or $4c$) represents the number of adults on the bus before the stop. When 15 adults get off at the bus stop there are now $4c – 15$ adults on the bus. There are still c children on the bus. The new ratio of adults to children is $4c – 15$ to c or $\dfrac{4c - 15}{c}$. The information given states that after the stop the ratio of adults to children becomes 5 to 2. These two ratios represent the same relationship when comparing the number of adults to the number of children. The two ratios can be set equal to each other. $\dfrac{4c - 15}{c} = \dfrac{5}{2}$.

Incorrect Response B: This response reverses the relationship between the adults and children. There are 4 adults for each child. To make an equation where the numerical value is the same, four times the number of children is needed. For example, if there are 20 adults and 5 children, $4a = c$ would be $4 \bullet 20 = 5$, which is not true. However, $a = 4c$ would be $20 = 4 \bullet 5$, which is true.

Incorrect Response C: This response does not keep the comparison consistent: $\dfrac{c}{4c - 15}$ is the ratio of children compared to adults; but the right half of the equation is 5 over 2, which is the ratio of adults to children. A proportion consists of two ratios that are equal, each representing the same numerical relationship between adults and children.

Incorrect Response D: As in response C, this response is not consistent in representing the order of comparison, and also makes the error discussed in response B.

24. The sum of four consecutive integers is greater than 25, but less than 50. If x represents the least of the four integers, which of the following inequalities can be used to solve for x?

    A. $25 < 4x + 4 < 50$

    B. $25 > 4x + 4 < 50$

    C. $25 < 4x + 6 < 50$

    D. $25 > 4x + 6 < 50$

Correct Response C: Consecutive integers go up by one each time. If x represents the least (lowest) of the four integers, then $x + 1$ is the second integer, $x + 2$ is the third integer (adding 1 to $x + 1$), and $x + 3$ is the fourth integer (one more than $x + 2$). The sum is the answer to an addition problem. The sum of four consecutive integers would be $x + (x + 1) + (x + 2) + (x + 3)$. The parentheses are not necessary here, but help to "show" each individual integer. Addition is commutative: numbers may be added in any order.

$$x + (x + 1) + (x + 2) + (x + 3) = x + x +1 + x + 2 + x + 3 = x + x + x + x + 1 + 2 +3 = 4x +6$$

The problem states that the sum, $4x + 6$, must fall between 25 and 50. The symbol ">" means "greater than." The symbol "<" means "less than." If the sum is greater than 25, then 25 is "less than" the sum and the sum is "less than" 50. Symbolically, that is: $25 < 4x + 6 < 50$.

Incorrect Response A: This response may come from interpreting adding four consecutive integers as adding $x + 1$ four times. The inequalities are used correctly.

Incorrect Response B: This response includes the same error as response A, but also reverses the left inequality. This would read "25 is greater than the sum of the four consecutive numbers" instead of "25 is less than the sum of the four consecutive numbers."

Incorrect Response D: This response "reverses" the left inequality, which is one of the errors made in response B.

25. At a constant temperature, the volume of a gas is inversely proportional to the pressure exerted on the gas. The volume of a certain gas sample is 12 cubic feet at a pressure of 1000 millimeters of mercury. What will the volume of this gas sample be at a pressure of 600 millimeters of mercury if the temperature is kept constant?

    A. 7.2 cubic feet

    B. 20 cubic feet

    C. 72 cubic feet

    D. 200 cubic feet

Correct Response B: If a relationship is inversely proportional, the product of the quantities involved is constant. If one factor gets larger, then the other factor must get smaller to keep the product the same. For example, suppose $a \bullet b = 40$, then if $a = 1$, $b = 40$; if $a = 2$, $b = 20$; if $a = 4$, $b = 10$; if $a = 5$, $b = 8$; and so on . . . The 40 is called the constant of variation. If $a \bullet b = 40$, then $a = \frac{40}{b}$. This form is commonly used to express inversely proportional quantities.

If V is inversely proportional to P, then $V = \frac{c}{P}$ or $V \bullet P = c$, where c is the constant of variation. V is the volume of gas and P is the pressure exerted on the gas. When $V = 12$, it is given that $P = 1000$. Then $c = V \bullet P = 12 \bullet 1000 = 12000$ in this situation. If $V \bullet P = 12000$ and $P = 600$ in the problem given, then $V \bullet 600 = 12000$, and $V = 20$.

Incorrect Response A: This response uses a direct proportion to attempt to solve the problem. In a direct proportion, as one part increases the other part also increases, and as one part decreases the other part also decreases, always in a proportional way. In this case, one quantity is multiplied by the constant of variation to get the second quantity: $a = c \bullet b$ instead of $c = a \bullet b$, where c is the constant.

Incorrect Response C: This response uses a direct proportion similar to response A and also makes an error of factor 10 in the multiplication or the division.

Incorrect Response D: This response likely uses the correct inverse proportional relationship but makes an error of factor 10 in the multiplication or division.

26. Which of the following ordered pairs would lie on the graph of the function $f(x) = 2|x| – 1$?

    A. (–7, –3)

    B. (–3, –7)

    C. (–3, 5)

    D. (5, –3)

Correct Response C: Each of the pairs given are called ordered pairs. The first member is the *x*-value followed by a comma and then the *y*-value: (*x*, *y*). Sometimes, function notation is used to denote the "answer" that results when a number is substituted for *x*. To find *f*(2), for example, 2 is substituted for *x*: $f(2) = 2\,|2| – 1 = 2 \bullet 2 – 1 = 3$. So $f(2) = 3$ and the ordered pair is written (2, 3). If a point lies on the graph of a function, its coordinates must "satisfy" the equation/function. They must make the function equation a true statement. Use each pair in the function to see which pair satisfies the equation. This will find the correct response by elimination. This function contains absolute value. Absolute value is the distance an amount is from zero, regardless of direction. It is always a value of zero or above since distance is a positive value. $|4| = |–4| = 4$ because both 4 and -4 are a distance of 4 units from zero.

Given (–3, 5): $f(–3) = 2|–3| – 1 = 2 \bullet 3 – 1 = 6 – 1 = 5$. $f(–3) = 5$. When –3 is substituted for *x*, 5 is the result.

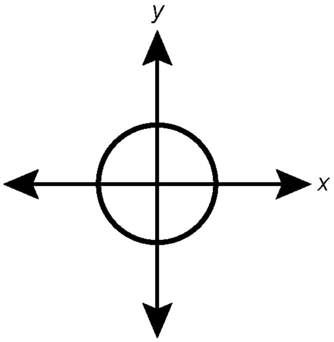
Incorrect Response A: This response may be the result of reading the ordered pair backward, substituting –3 for *x* in the equation, and ignoring the absolute value to get a result of –7.

Incorrect Response B: This response comes from reading the ordered pair correctly, ignoring the absolute value, and using –3 instead of 3 in the computation: $(2 \bullet –3) –1 = –7$.

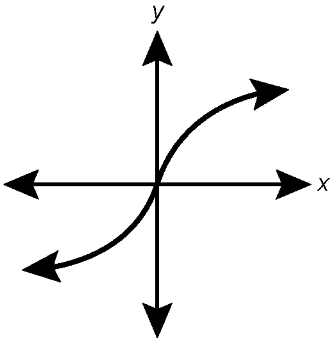
Incorrect Response D: This response could be the result of reading the ordered pair backward, substituting –3 for *x* in the equation, and performing the computation correctly.

27. Which of the following graphs represents y as a function of x?

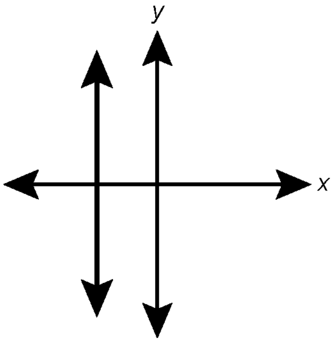
A.



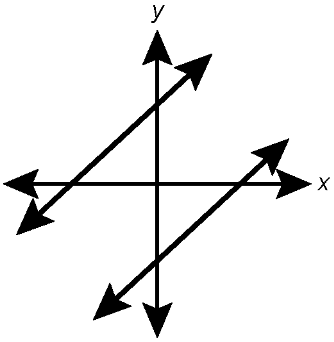
B.



C.



D.



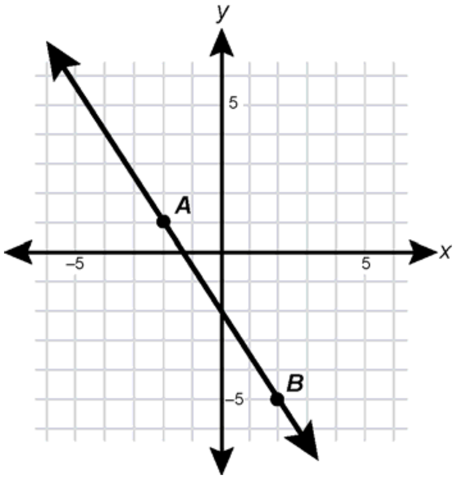
Correct Response B: A function is a special subset of equations for which every input (*x*) has one and only one output (*y*). Imagined lines drawn parallel to the *y*-axis will only intersect the graph once if it is a function. Graph B has exactly one *y*-value for each of its *x*-values.

Incorrect Response A: To show that a graph does not represent a function, it is necessary to show that at least one *x*-value is paired with more than one *y*-value. There are many *x*-values on this graph that are paired with two *y*-values. For example, where $x = 0$ there are two *y*-values, one above zero and one below zero on the *y*-axis.

Incorrect Response C: Again, it is necessary to show that at least one *x*-value is paired with more than one *y*-value. This graph is a line parallel to the *y*-axis. For one value of *x*, there are an infinite number of *y*-values. For example, if $x = –6$, pairs on the graph could be (–6, –8), (–6, –2), (–6, 0), (–6, 1), (–6, 2.5), etc.

Incorrect Response D: Again, it is necessary to show that at least one *x*-value is paired with more than one *y*-value. On this graph, for every *x*-value there are two *y*-values, one on each of the two parallel lines. For example, at $x = 0$ there is a point on the top line intersecting the *y*-axis and a point on the bottom line intersecting the *y*-axis. At $x = 0$, *y* can equal both of these values.

28. **Use the graph below to answer the question that follows.**



Which of the following lines, if graphed on the coordinate system above, would be parallel to line *AB*?

A. $y = 3$

B. $y = 2x$

C. $y = -\frac{2}{3}x + 2$

D. $y = -\frac{3}{2}x + 1$

Correct Response D: Parallel lines on a graph have the same slope. Slope is the slant of the line. A steeper slant is a larger slope. If the slope is positive, the line slants from left to right going upward (↗). If the slope is negative, the line slants from left to right going downward (↘). Slope is the ratio of the rise to the run, or the amount of change in the vertical direction (↕) compared to the amount of change in the horizontal direction (↔). A visual way to find the slope is to start at point B on the given graph and move up and over to the *y*-intercept (where the graph crosses the *y*-axis). From point B, move UP 3 (i.e., +3) and LEFT 2 (i.e., –2). To get from the *y*-intercept to point A, move UP 3 units and LEFT 2 units. The slope is the ratio of vertical change to horizontal change, or $\frac{+3}{-2}$, which can also be written as $-\frac{3}{2}$. A straight line (linear) graph represents a constant or steady rate. Any two points chosen to find the slope will simplify to the same ratio.

Another approach is to use the formula for slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$, where *m* = the slope of the line between two points $A(x_1,y_1)$ and $B(x_2,y_2)$. On the given graph, *A* is (–2, 1) and B is (2, –5). The *y*-intercept (0, –2) could also have been used as one of the two points instead of *A* or *B*. Any easily read point on the graph should work. Using *A* and *B*, $m = \frac{-5-1}{2-(-2)} = \frac{-6}{4} = -\frac{3}{2}$. If the *y*-intercept was used with point *A*, then $m = \frac{-2-1}{0-(-2)} = -\frac{3}{2}$.

Compare this slope to the slopes of the given equations. All of the equations are in slope-intercept form. Slope-intercept form is $y = mx + b$, where *m* is the slope and *b* is the y-intercept. The coefficient of *x* is the slope of each of the lines. Response D has a slope of $-\frac{3}{2}$.

Incorrect Response A: $y = 3$ can be written in slope-intercept form as $y = 0x + 3$. The coefficient of *x* is the slope of the line. A zero slope is flat. This equation will have a graph with a line that is parallel to the *x*-axis and with a *y*-intercept of 3.

Incorrect Response B: This graph would have a slope of 2 since the coefficient of the *x* is 2. The graph will slant upward from left to right, so it will intersect the given graph.

Incorrect Response C: This response is most likely a result of calculating the slope as the change in *x* over the change in *y* rather than the change in *y* over the change in *x*. Slope is always vertical change compared to horizontal change.

29. **Use the table below to answer the question that follows.**

Depth Below Surface (kilometers)	Temperature (°C)
5	80
9	180

In 1984 a deep drilling project showed that for depths greater than 3 kilometers, the temperature of the earth's crust can be modeled as a linear function of the depth below the surface of the earth. Using a linear model and given the data in the table above, what is the temperature of the earth at a depth of 18 kilometers?

A. 315°C

B. 360°C

C. 405°C

D. 430°C

Correct Response C: Linear functions have a constant rate of change. This is the slope of the line. If two points are given, the rate of change or slope can be found. Slope = the change in *y*-values between two points compared to the change in *x*-values between the same two points. Written as a formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$. From the table, P_1 = (5, 80) and P_2 = (9, 180), so the rate of change is $\frac{180 - 80}{9 - 5}$ = 25 or 25 to 1. This means that there is a change in temperature of 25°C for each kilometer of depth; for every one kilometer increase in depth, the temperature rises 25°C.

The question asks what the temperature will be at a depth of 18km. The depth increases another 9 kilometers beyond the deepest value in the table. For each of those 9 extra kilometers, the temperature goes up 25 degrees. 9 • 25 is 225 degrees. At 9 kilometers, the temperature was 180 degrees. 180°+ 225°= 405°.

Another approach to the problem is to find the equation that represents the relationship between the temperature and the depth. It is a linear relationship according to the information given. One form of the linear equation is the slope-intercept form: $y = mx + b$, where m is the slope and b is the y-intercept. In this case, the y is the temperature and the x is the depth. The slope of the line was found above: m = 25, so the equation has the form $y = 25x + b$. Two points on the graph are given in the table. They are on the graph and must "satisfy" the equation (i.e., make the equation true). Using (5, 80), the 5 is substituted for x and the 80 is substituted for y: 80 = 25 • 5 + b, 80 = 125 + b, b = –45. The equation is $y = 25x – 45$. If (9, 180) was used instead, the result would have been the same since both points are on the line and must work in the equation. With the equation found, if x = 18, then y = 25 • 18 – 45 = 450 – 45 = 405.

Incorrect Response A: This response might have come from incorrectly assuming a rate of change using the first data point: 80 degrees change for every 5 kilometers. Going from 5 km to 9 km takes 4 units while going from 80° to 180° takes 100 units, so there is an increase of 25° per unit. Putting those two ideas together: 18 = 3 groups of 5 plus 3 extra units so the new temperature = 3 × 80 + 3 × 25.

Incorrect Response B: This response might have come from failing to consider the rate as the change in temperature compared to the change in depth, using only the second data point and thinking that every 9 km results in a change of 180 degrees.

Incorrect Response D: This response might have come from seeing that for every 4 km the temperature changed by 100 degrees but incorrectly adding 50 degrees for the last one km of change in depth instead of adding 25 degrees.

30. On a coordinate plane, the graph of the function $y = mx + b$ has a positive y-intercept and a negative x-intercept. Which of the following statements must be true?

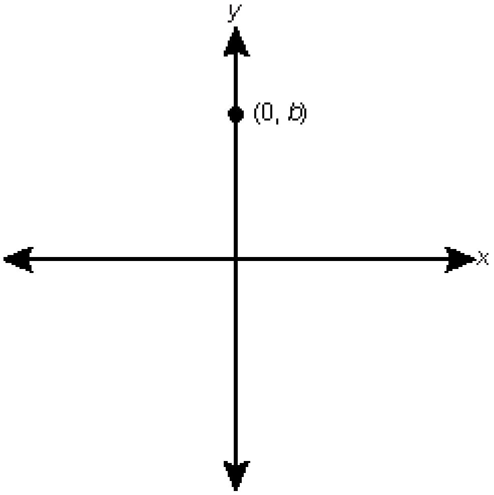
    A. $m > 0$ and $b > 0$

    B. $m > 0$ and $b < 0$

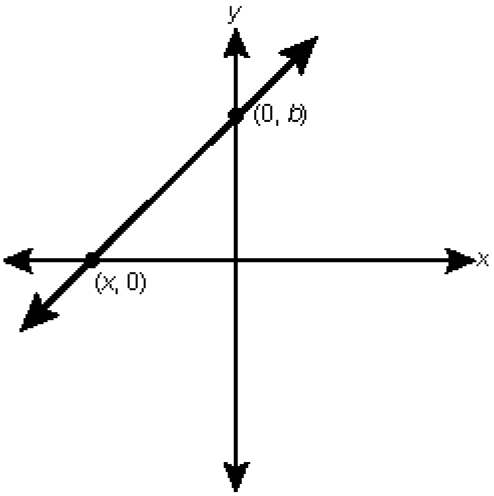
    C. $m < 0$ and $b > 0$

    D. $m < 0$ and $b < 0$

Correct Response A: $y = mx + b$ is the slope-intercept form of a linear equation. A linear equation has a graph that is a straight line. The *m* represents the slope or slant of the line. The *b* is the y-intercept. A positive *y*-intercept means that the intercept is on the *y*-axis above zero.



A negative *x*-intercept means that the graph crosses the *x*-axis to the left of zero. Connecting the two intercepts gives a sketch of the graph.



It is given that the *y*-intercept (*b*) is positive so $b > 0$. The slope, *m*, of the line is also positive if the line rises from left to right. To move from the *x*-intercept (*x*, 0) to the *y*-intercept (0, *y*), go UP and RIGHT. Both moves are positive directions on the graph, so $m > 0$ also.

Incorrect Response B: This response correctly identifies the slope as positive but incorrectly identifies the *x*-intercept value as *b*.

Incorrect Response C: This response incorrectly identifies the slope as negative but correctly identifies the *y*-intercept value as *b*.

Incorrect Response D: This response incorrectly identifies both the slope and *y*-intercept.

31. In the early morning a baker sells 2 dozen muffins, and no doughnuts. For the rest of the day, sales follow a pattern of 7 doughnuts for every 2 muffins. If the horizontal axis (x-axis) represents the number of doughnuts sold in a day and the vertical axis (y-axis) represents the number of muffins sold in a day, which of the following statements describes the graph of a line drawn through the data points?

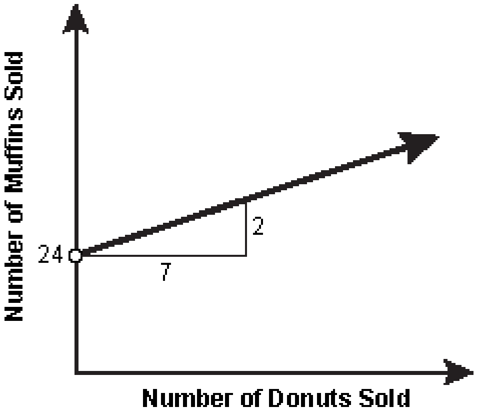
    A. The line has an x-intercept at 24 and a slope of $\frac{7}{2}$.

    B. The line has an x-intercept at 24 and a slope of $\frac{2}{7}$.

    C. The line has a y-intercept at 24 and a slope of $\frac{7}{2}$.

    D. The line has a y-intercept at 24 and a slope of $\frac{2}{7}$.

Correct Response D: $y = mx + b$ is the slope-intercept form of a linear equation. A linear equation has a straight line for its graph. The *m* value is the slope or slant of the line. Slope is the rate of change, or the ratio of how much the *y*-values change compared to how much the *x*-values change. The *b* value is where the graph crosses the *y*-axis. It is called the *y*-intercept of the graph. In this situation, the *y*-axis represents the number of muffins sold and the *x*-axis represents the number of doughnuts sold. The pattern is 7 doughnuts sold for every 2 muffins or 2 muffins sold for every 7 doughnuts. Slope is the ratio of change in *y* to the change in *x*, so the ratio of the number of muffins sold to the number of doughnuts sold is the slope: $m = \frac{2}{7}$. Before this pattern begins, two dozen (or 24) muffins are sold. At 0 doughnuts sold, 24 muffins have already been sold. This is the point (0, 24) on the graph, and is also the *y*-intercept of the graph.



Incorrect Response A: This response comes from incorrectly applying the definition of slope, comparing the change in *x*-values to the change in *y*-values instead of the other way around. Alternatively, this response could have come from reversing the axes, using the *x*-axis to represent the number of muffins sold and the *y*-axis to represent the number of doughnuts sold.

Incorrect Response B: This response correctly identifies the slope but misinterprets (0, 24) as the *x*-intercept.

Incorrect Response C: This response interprets (0, 24) correctly as the *y*-intercept but incorrectly applies the definition of slope as in response A.

32. The length, width, and height of a rectangular box are each doubled. How does the surface area of the larger box compare to the surface area of the original box?

    A. 2 times the surface area of the original box

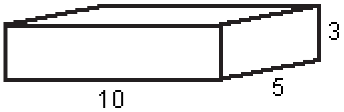
    B. 4 times the surface area of the original box

    C. 12 times the surface area of the original box

    D. 24 times the surface area of the original box

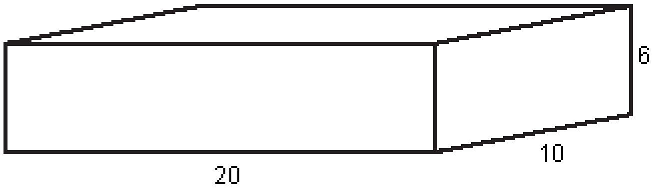
Correct Response B: Surface area (SA) is the sum of the areas of all six sides of the box (top + bottom + left side + right side + front + back). Two approaches will work to solve this. One approach is to do a specific example and compare the results.

Let length = 10, width = 5, height = 3.



All six sides are rectangles. Area of a rectangle = length x width. Front and back are each *l* • *h* = 10 • 3 = 30. Top and bottom are each *l* • *w* = 10 • 5 = 50. Left and right sides are each *w* • *h* = 5 • 3 = 15. SA = 2 • 30 + 2 • 50 + 2 • 15 = 60 + 150 + 30. The surface area is 190 square units.

Now double each of the dimensions and find the new surface area. Let *l* = 20, *w* = 10, *h* = 6.



Front and back are each 20 • 6 = 120. Top and bottom are each 20 • 10 =200. Left and right sides are each 10 • 6 = 60. SA = 2 • 120 + 2 • 200 + 2 • 60 = 240 + 400 + 120. The new surface area is 760 square units.

Compare the new area to the original area. 760 to 190 simplifies to 4 to 1. The new area is four times as large as the original area.

Another approach is a general approach using algebra. The original surface area is represented by SA = 2*lw* + 2*wh* + 2*lh*. Each dimension (*l*, *w*, and *h*) must be doubled. Replace *l* with 2*l*, *w* by 2*w* and *h* by 2*h*. SA = 2(2*l*)(2*w*) + 2(2*w*)(2*h*) + 2(2*l*)(2*h*) = 2(4*lw*) + 2(4*wh*) + 2(4*lh*) =8*lw* + 8w*h* + 8*lh*. Compare the result to the original surface area: SA = 2*lw* + 2*wh* + 2*lh*. If the original equation is multiplied by 4, the new equation is derived. The new area is four times as large as the original area.

Incorrect Response A: This response results from a common misconception. Doubling the surface area could be the result of only one of the dimensions being doubled. For example, *l* • (2*w*) = 2(*lw*). Since area is the product of two dimensions and each one is doubled, the product is always four times larger: 2*l* • 2*w* is 4(*lw*).

Incorrect Response C: This response might come from assuming that each face of the larger box has twice the area of the original face. There are six faces, so two times the six equals 12.

Incorrect Response D: This response might come from correctly comparing the area of one face, *l* • *w*, to the area of that face when lengths are doubled, 2*l* • 2w = 4*lw*, then incorrectly multiplying this result by six for the six faces: 6 • 4*lw* = 24*lw*.

33. **Use the information below to answer the question that follows.**

1 kilometer = 1000 meters
1 hour = 3600 seconds

Mach numbers were invented to report the speeds of jet planes that can travel faster than the speed of sound. A Mach number tells how many times as fast as the speed of sound the jet travels. For example, Mach 2 is twice the speed of sound.

A jet travels at Mach 3. If the speed of sound in air is approximately 344 meters per second, which of the following expressions represents the speed of the jet in kilometers per hour?

A. $\frac{3 \times 344 \times 3600}{1000}$

B. $\frac{3 \times 344 \times 1000}{3600}$

C. $\frac{344 \times 3600}{3 \times 1000}$

D. $\frac{344 \times 1000}{3 \times 3600}$

Correct Response A: Mach 3 is three times the speed of sound. The speed of sound is 344 meters per second. Multiplying the speed of sound by 3 yields the speed of the jet per second: (3 x 344). The speed of the jet per hour is found by multiplying meters per second times seconds per hour. There are 3600 seconds in an hour. For each of the 3600 seconds that the jet travels, it covers 3 × 344 meters. Therefore, (3 × 344) × 3600 is the number of meters the jet goes in one hour. To convert the speed from meters to kilometers, divide by 1000. There are 1000 meters in each kilometer. Each group of 1000 meters is a kilometer. The number of groups of 1000 in (3 × 344) × 3600 will be the speed in kilometers per hour that the jet travels. The solution is: $\frac{(3 \times 344) \times 3600}{1000}$.

Incorrect Response B: This response comes from doing the reverse of what is needed. It multiplies by 1000 and divides by 3600.

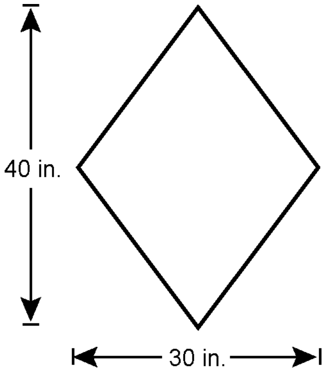
Incorrect Response C: This response comes from incorrectly using/interpreting the meaning of Mach 3. Three times the speed of sound is the speed of the jet.

Incorrect Response D: This response makes both errors discussed for responses B and C.

34. **Use the statements and the diagram below to answer the question that follows.**

The diagonals of a rhombus bisect each other.

The diagonals of a rhombus are perpendicular to each other.



A window is being manufactured in the shape of the rhombus shown above. Its height will be 40 inches and its width will be 30 inches. If the window is to have a single pane of glass, how many square inches of glass will be required?

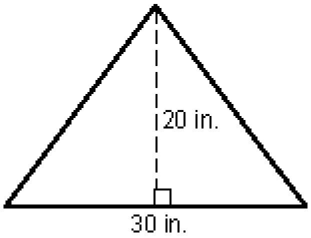
A. 150

B. 300

C. 600

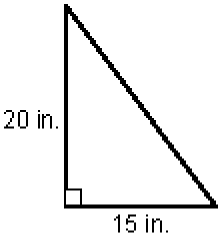
D. 1200

Correct Response C: A rhombus is a parallelogram with all sides equal. A parallelogram is a four-sided figure with opposite sides parallel. Several approaches will find the solution. One approach is to look at half of the figure as a triangle with a base of 30 inches.



The information given says that the diagonals of the original figure bisect (cut into two equal parts) each other and are perpendicular (form a 90°angle) to each other. The area of a triangle is $\frac{1}{2}bh$ or half the product of the base and the height. The triangle above has a 30-inch diagonal as its base, and half of the 40-inch diagonal (40 ÷ 2 = 20) for its height. $\frac{1}{2}bh = \frac{1}{2} \bullet 30 \bullet 20 = \frac{1}{2} \bullet 600 = 300$ square inches. This area represents half of the area of the glass so it must be doubled: 2 • 300 = 600 square inches.

Another approach looks at one-fourth of the rhombus, a right triangle.



$A = \frac{1}{2}bh = \frac{1}{2} \bullet 15 \bullet 20 = \frac{1}{2} \bullet 300 = 150$ square inches. Note that the base is half of the 30-inch diagonal this time. There are four of these right triangles in the pane of glass so 150 must be multiplied by four to find the total area: 4 • 150 = 600 square inches.

Incorrect Response A: This response comes from correctly finding the area of the right triangle, which represents one-fourth of the rhombus, but then forgetting to multiply by 4 to find the total area.

Incorrect Response B: This response comes from finding the area of half of the rhombus, but then not doubling it to find the area of the entire pane of glass.

Incorrect Response D: This response might come from forgetting to multiply one-half of the base times height when finding triangle area, or incorrectly thinking that the area is the product of the given measurements.

35. To measure the length of a winding path through a city park, a park manager rolled an instrument consisting of a wheel with a 10-inch radius along the path. The wheel made 200 revolutions. If 3 is used as an estimate for π, approximately how long is the path in feet?

A. 500 feet

B. 1,000 feet

C. 5,000 feet

D. 12,000 feet

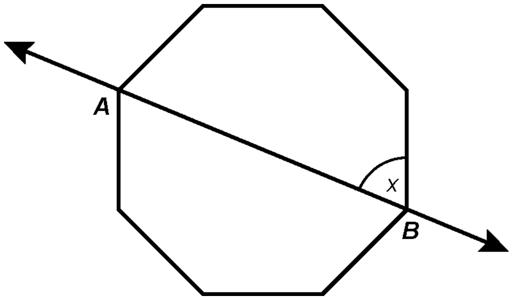
Correct Response B: The outside of the wheel made 200 revolutions. Each revolution is the circumference of the wheel. Circumference is the distance around the outside of the circle. $C = \pi d$ (or, circumference equals pi times the diameter). The diameter of a circle fits around its circumference a little over 3 times (3.14 is usually used, but 3 as an approximation is given here). The radius of the wheel is 10 inches. The radius is the distance from the center of the circle to circumference of the circle. The diameter is the distance across a circle through its center. The diameter is the length of two radii. If the radius is 10 inches, then the diameter is 20 inches. $C = \pi d = 3 \bullet 20 = 60$ inches. Each complete revolution of the wheel covers 60 inches. Every 12 inches equals one foot. $60 \div 12 = 5$, so each revolution of the wheel covers 5 feet. The wheel rotates 200 times. 200 × 5 feet is 1000 feet.

Incorrect Response A: This response uses the radius instead of the diameter to try to find the circumference. The correct formula would then be $C = 2\pi r$, not $C = \pi r$.

Incorrect Response C: This response uses the area formula, $A = \pi r^2$, instead of the formula for the circumference.

Incorrect Response D: This response gives the answer in inches, rather than dividing by 12 to convert the units to feet.

36. **Use the diagram below to answer the question that follows.**



Line *AB* is one of the lines of symmetry in the regular octagon shown above. What is the measure of angle *x*?

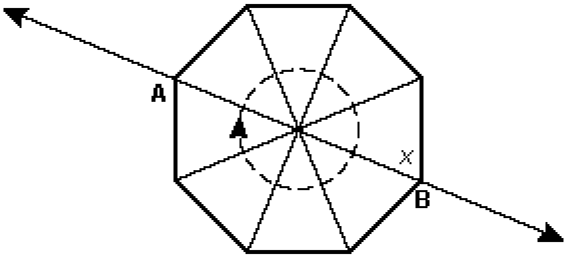
A. 45°

B. 56.25°

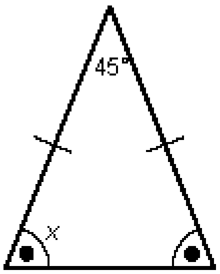
C. 60°

D. 67.5°

Correct Response D: There are several ways to see and approach this problem. One approach is to cut the octagon into 8 congruent triangles.

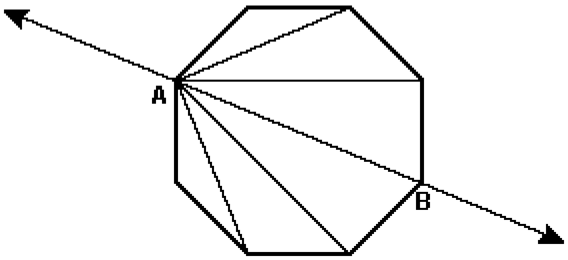


The center of the octagon is common to all 8 of the triangles. The angles formed are called central angles and are all equal. There is a total of 360° in the 8 central angles, since they form a circle. 360 ÷ 8 = 45, so each of those central angles is 45°.



Each of these triangles is an isosceles triangle. An isosceles triangle is a triangle with two equal sides. Each of the two sides goes from the center to a vertex. A stop sign is a regular figure, meaning all the sides are equal and all the angles are equal, so the distance from the center to each vertex will be the same. The two base angles of the triangle will be equal since the opposite sides are equal. The angles in a triangle add up to 180 degrees. The top angle in the triangle is 45°. 180°– 45° = 135° and the two base angles are equal, so half of 135 will be the measure of each one: 135° ÷ 2 = 67.5°. x is the measure of one of the base angles so $x = 67.5°$.

Another approach is to cut the octagon into triangles using points *A* and *B*.



There are six triangles formed. The angles in each of the triangles add up to 180°. There are six triangles for a total of 6 • 180 = 1080°. Notice that all of the angles in the triangles are pieces of the angles in the octagon. Since all of the angles in a **regular** octagon are equal, 1080 ÷ 8 = 135° gives the size of each angle in the octagon. Because $\overleftrightarrow{AB}$ is a line of symmetry, the line segment $\overline{AB}$ bisects the octagon and bisects angle *B*. Angle *x* equals half of angle *B*: 135° ÷ 2 = 67.5°.

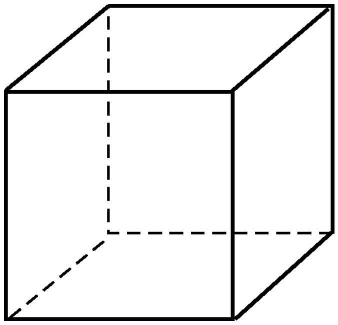
This second approach leads to the formula for finding the size of each angle in any **regular** polygon: $angle\ \ measure\ = \frac{180\,(n-2)}{n}$, where n is the number of sides in the polygon. In this case $n = 8$ since there are 8 sides. $\frac{180\,(8-2)}{8}$ gives the result of 135° for each angle.

Incorrect Response A: This response comes from correctly finding the value of the central angles in the octagon, but incorrectly assuming that the other two angles in each of the triangles are the same as the central angle. The three angles in a triangle always add up to 180°. 3 × 45° = 135°, which is not enough.

Incorrect Response B: This response might come from incorrectly using the above formula to find the total number of degrees in the angles of an octagon, calculating $180(n – 3)$ instead of $180(n – 2)$.

Incorrect Response C: This response might come from assuming that each triangle is equilateral (where all sides are equal), and thus 180° in a triangle ÷ 3 = 60°. The distance from the center to each vertex is the same but the octagon sides, while equal to each other, could be any length.

37. **Use the diagram below to answer the question that follows.**



Euler's polyhedron formula relates the numbers of faces, edges, and vertices of any closed polyhedron. If E, F, and V represent the number of edges, faces, and vertices respectively of the cube shown above, which of the following equations relating these values is true?

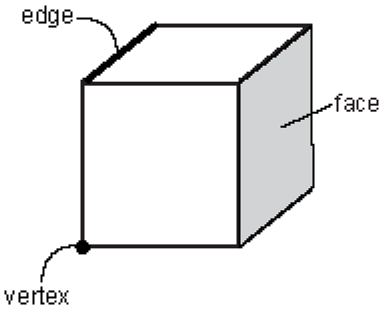
A. $F + V = E + 2$

B. $F + V = E + 6$

C. $F + V = E + 8$

D. $F + V = E + 14$

Correct Response A:



A cube has six faces: top and bottom, back and front, and left and right sides. A cube has eight vertices (corners) where 3 faces meet. There are four on the top and four on the bottom. A cube has 12 edges (where two faces meet). Four edges are on the top, four edges are on the bottom, and there are four edges "connecting" the top to the bottom. So, $F = 6$, $V = 8$, and $E = 12$.

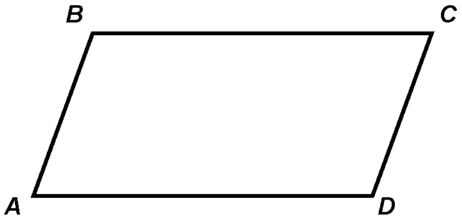
The easiest way to find which formula is correct is to put these numbers into each one and see which one is true. $F + V = E + 2$ is $6 + 8 = 12 + 2$, which is true: $14 = 14$.

Incorrect Response B: $F + V = E + 6$ is $6 + 8 = 12 + 6$, which is false: $14 \neq 18$.

Incorrect Response C: $F + V = E + 8$ is $6 + 8 = 12 + 8$, which is false: $14 \neq 20$.

Incorrect Response D: $F + V = E + 14$ is $6 + 8 = 12 + 14$, which is not true: $14 \neq 26$.

38. **Use the diagram below to answer the question that follows.**



Quadrilateral *ABCD* is a parallelogram. Which of the following statements is true?

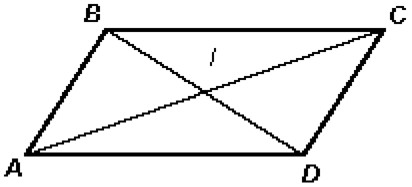
A. *ABCD* has exactly two lines of symmetry.

B. Point *C* is the reflection of point *A* across diagonal *BD*.

C. Reflecting *ABCD* over side *CD* is equivalent to a horizontal translation of length *BC*.

D. *ABCD* has 180° rotational symmetry about the point of intersection of its diagonals.

Correct Response D: When diagonals $\overline{AC}$ and $\overline{BD}$ are drawn, they intersect at *I*.

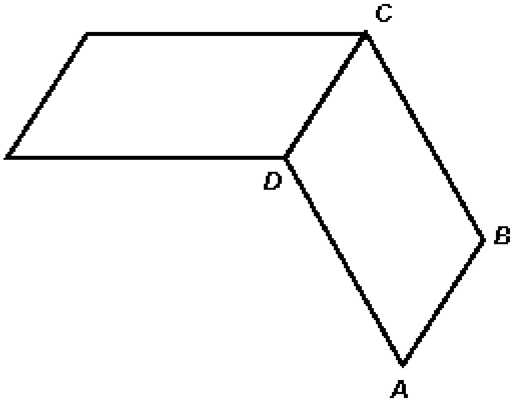


If the parallelogram is turned 180° around point *I*, *C* falls where *A* had been, and *B* falls where *D* had been, etc. This is rotational symmetry. It will look the same as it did before the rotation except for the labeling.

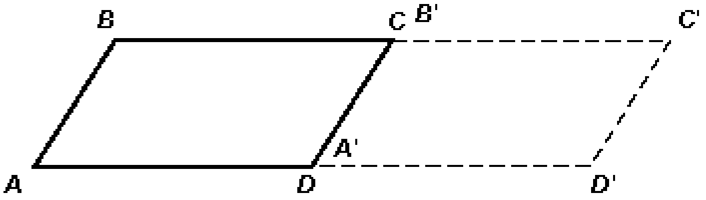
Incorrect Response A: A parallelogram has no lines of symmetry. When folded on a line of symmetry, the two halves of a figure line up with each other. No matter which line it is folded on, the two halves of a parallelogram will not line up because neither the adjacent sides nor the adjacent angles are the same size.

Incorrect Response B: If the parallelogram is folded along diagonal $\overline{BD}$, point *A* does not land on point *C*. This can only happen if the diagonals intersect at a 90° angle (i.e., are perpendicular).

Incorrect Response C: If parallelogram *ABCD* is reflected over $\overline{CD}$, the $\overline{CD}$ side is fixed and does not move. The result will be:

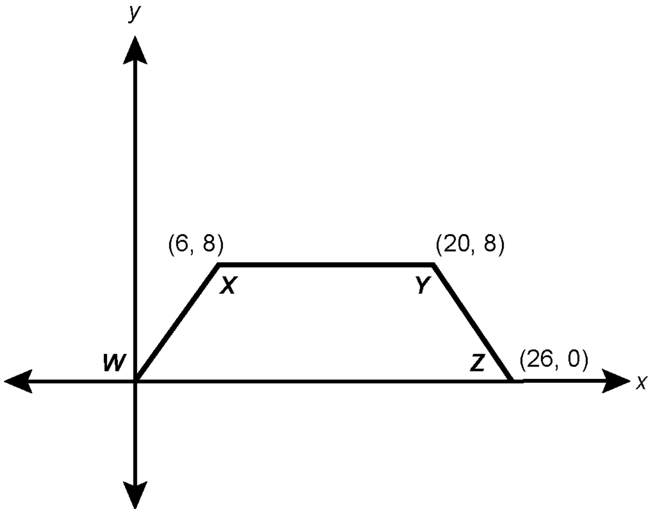


A translation is a slide horizontally or vertically, or both. A translation to the right of point *B* the length of $\overline{BC}$ would give:



The easiest way to explore this would be to cut out a model of a parallelogram and try each of the situations given.

39. **Use the graph below to answer the question that follows.**



What is the perimeter of quadrilateral *WXYZ* in the graph above?

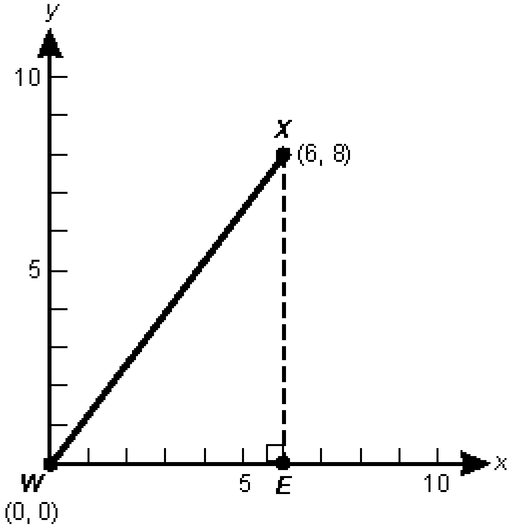
A. 52

B. 58

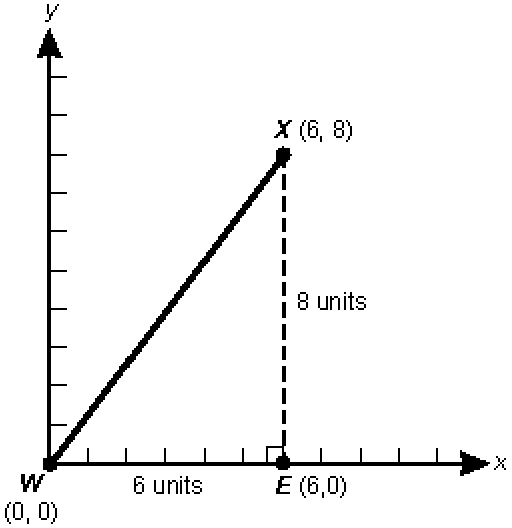
C. 60

D. 62

Correct Response C: *WZ* = 26 units. It is on the *x*-axis with endpoints (0, 0) and (26, 0), so the length is 26 – 0. *XY* = 14 units. Notice that the *y*-values of point *X* (6, 8) and point *Y* (20, 8) are the same. There is no up or down change. The segment is "flat." The distance or length from 6 to 20 is 14 units (20 – 6). $\overline{WX}$ is on a slant, from *W* (0, 0) to *X* (6, 8). This forms the hypotenuse of a right triangle if a perpendicular line segment is drawn from point *X* to the *x*-axis. Point *E* has the same *x*-value as point *X* because it is the same distance horizontally from zero.

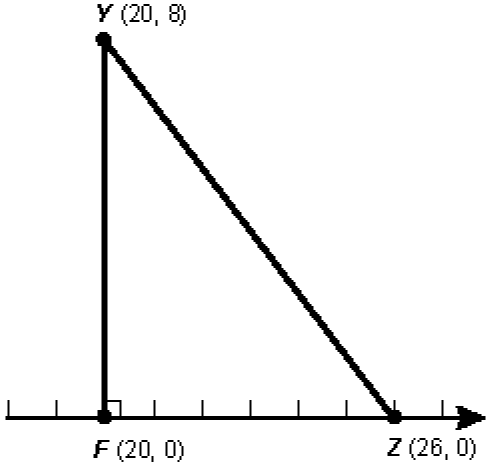


XE = 6 units because point *E* is directly below point *X*. The distance from point *E* up to point *X* is 8 units.



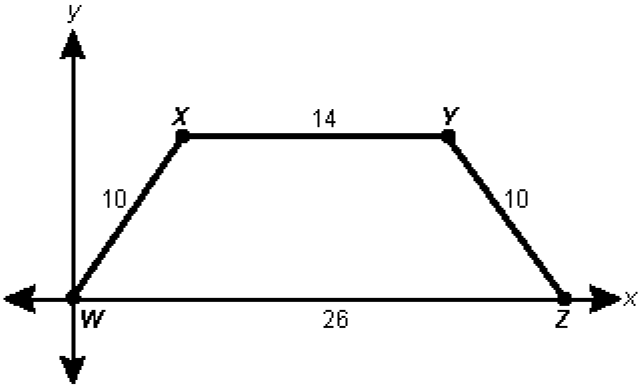
This is a right triangle, so the Pythagorean Theorem may be used: $a^2 + b^2 = c^2$ where *a* and *b* are the sides forming the right angle and *c* is the hypotenuse (the side opposite the right angle). In the diagram above, $a = 6$ and $b = 8$, so $6^2 + 8^2 = c^2$. Because 36 + 64 = 100, $c^2 = 100$ and $c = 10$ units. The distance from *W* to *X* is 10 units.

The same procedure can be used to find the length of $\overline{YZ}$.



Point *F* is directly below point *Y*, so it will have the same *x*-coordinate of 20. It is on the *x*-axis, so its *y*-coordinate will be zero: *F* (20, 0). The distance on the *x*-axis from 20 to 26 is 6 units. *FZ* = 6 units. Point *Y* is 8 units directly above point *F*, so *YF* = 8 units. These are the same values as the ones used for triangle *WXE*, so *YZ* = 10 units also.

Perimeter is the distance around a figure. It is the total of the lengths of all of the sides.



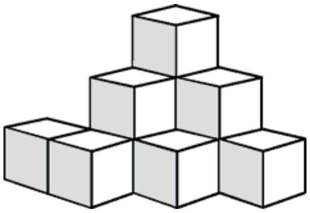
Perimeter = 10 + 14 + 10 + 26 = 60 units.

Incorrect Response A: This response correctly finds the lengths of $\overline{WZ}$ and $\overline{XY}$, but then uses the *x*-coordinate of point *X* as the lengths of $\overline{WX}$ and $\overline{YZ}$: 14 + 26 + 6 + 6 = 52 units.

Incorrect Response B: This response correctly finds the length of $\overline{WZ}$, but incorrectly uses the *x*-coordinate of point *Y* as the length of $\overline{XY}$, not noticing that it is shorter than 20 units since its left end is not at zero. This response also contains the same error as response A, using 6 for the length of $\overline{WX}$ and $\overline{YZ}$: 26 + 20 + 6 + 6 = 58.

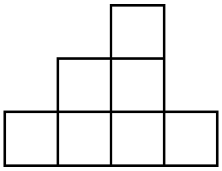
Incorrect Response D: This response correctly finds the length of $\overline{WZ}$, correctly uses *XY* = 20 units as in response B, and then incorrectly uses the *y*-coordinate of point *X* as the lengths of $\overline{WX}$ and $\overline{YZ}$: 26 = 20 + 8 + 8 = 62.

40. **Use the diagram below to answer the question that follows.**

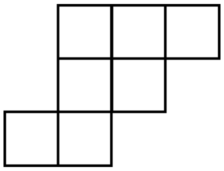


If the shaded faces in the figure shown above represent the front view of the figure, which of the following diagrams is the back view of the figure?

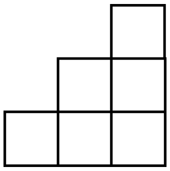
A.



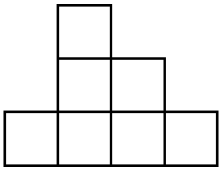
B.



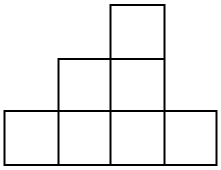
C.



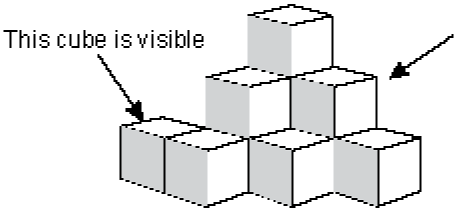
D.



Correct Response A:



The single cube seen on the far right from the front is on the left from the back, the stack of two cubes will be next, followed by the stack of three cubes. However, note that on the front view the far left cube sticks out past the stack of three cubes and on the backside will be visible.

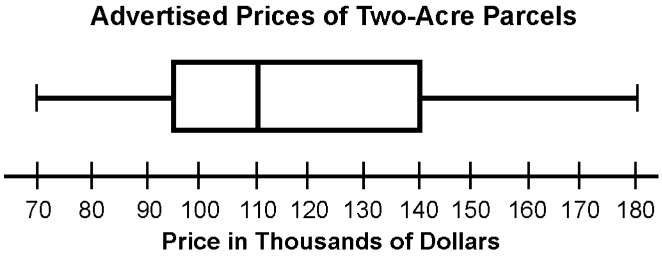


Incorrect Response B: This response shows the view from the top, looking down.

Incorrect Response C: This response could almost be the correct view but is missing the one cube from the far left of the front that can be seen from the back, or it could be the view from the back left instead of the back right.

Incorrect Response D: This response is the view from the front, ignoring that it would be reversed when looking at the figure from the back.

41. **Use the box plot below to answer the question that follows.**



The box plot above represents the advertised prices, in thousands of dollars, of 2-acre parcels of land in a particular county. Based on the box plot, which of the following statements is true?

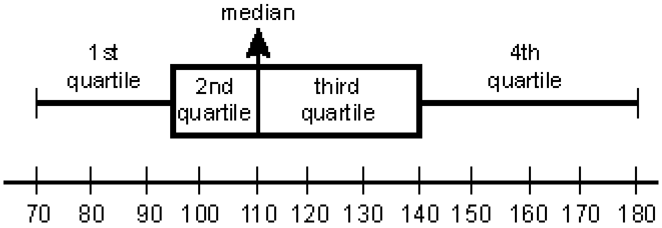
A. The mean price of a parcel is $110,000.

B. There is only one parcel priced at $180,000.

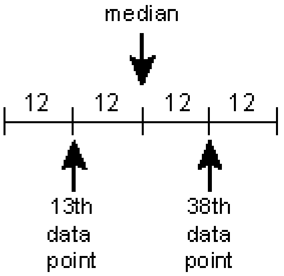
C. More than half of the parcels are priced higher than $110,000.

D. If the data represent 50 parcels, then at least 12 of them are priced at or below $95,000.

Correct Response D: This response correctly interprets the data. A box plot shows the range from lowest to highest of the parcel prices. In this case, the lowest asking price is $70,000 and the highest asking price is $180,000. The box "frames" the second and the third quartiles (fourths). The median value is the exact center of the graph. The "whiskers" show the first and fourth quartiles. The scale is in thousands of dollars, from 70 to 180. No information is given about how many of each price actually occurs.



In this case, when the cost of all 50 parcels of land are put in order, half of them are below the median and half are above. Since 50 is an even number, the median would be the average of the twenty-fifth and twenty-sixth asking price. For example, if there were nine prices, an odd number, the median would be the fifth price with four below and four above. If the prices are 20, 21, 23, 27, 30, 31, 35, 38, and 40, then the median is 30 in this example. If there are ten prices: 20, 21, 23, 25, 27, 30, 33, 35, 39, and 41, then the center is between the fifth and the sixth price (i.e., between 27 and 30). (27 + 30) ÷ 2 = 28.5, which is the median (the average of the two prices). That could be the scenario with fifty parcels. The median is not one of the 50 prices but rather is between the two prices in the center of the distribution. Twenty-five prices would be above and twenty-five would be below the median. There are 25 prices above the median, and the dividing line between the first and second quartiles is the 13th price, the median of these 25 prices. By similar reasoning, the dividing line between the third and fourth quartiles is the median of the 25 prices below the median of the entire distribution.

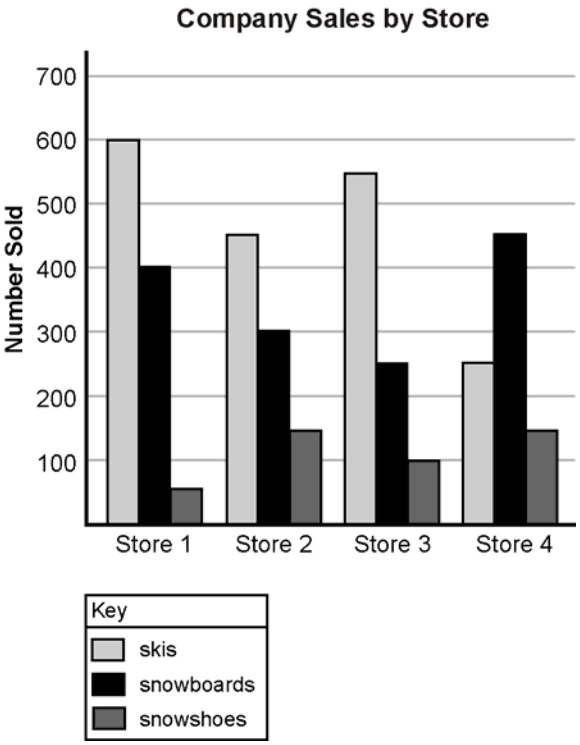


Incorrect Response A: The mean is not shown on a box and whisker plot. The $110,000 is the median price of the parcels.

Incorrect Response B: The plot shows the range in prices for the land but not how many there are of each. The $180,000 represents the highest price in the group but there could be several parcels at that price.

Incorrect Response C: $110,000 is the median price (the price that is exactly in the middle when all of the prices are arranged in numerical order). By definition there are the same number of parcels above and below the median.

42. **Use the information below to answer the question that follows.**



The data in the graph above represent the number of snowboards, pairs of skis, and pairs of snowshoes sold by a sporting goods company in each of four stores during the winter season. Which of the following circle graphs represents the sports equipment sold in Store 2?

A.

B.

C.

D.

Correct Response B: The graph shows that Store 2 sold approximately 150 pairs of snowshoes, 300 snowboards, and approximately 450 pairs of skis. This is a total of 900 pieces of winter sports equipment. The whole circle graph represents the 900. To find the most accurate circle graph, the fraction of the whole that each type of equipment represents is needed. There are 450 pairs of skis out of the 900 pieces of equipment: $\frac{450}{900} = \frac{1}{2}$. Since one-half of all the sales at Store 2 were skis, half of the circle graph must represent skis. 300 of the 900 pieces of equipment sold were snowboards: $\frac{300}{900} = \frac{1}{3}$. One third of all of the sales at Store 2 were snowboards. 150 of the 900 pieces of equipment sold were snowshoes: $\frac{150}{900} = \frac{1}{6}$. One sixth of all the equipment sold at Store 2 were snowshoes.

The fraction representing the three types of sports equipment must add up to the whole: $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3}{6} + \frac{2}{6} + \frac{1}{6} = \frac{6}{6} = 1$.

Notice that after finding the fractions that represent the skis, the problem is solved by elimination. Only one of the four circle graphs shows the ski sales represented by one-half of the graph.

To create the circle graph, cut the circle into six equal parts. Three of these parts equals one-half and represents the ski sales. Two of the equal parts represent the fraction of the whole that is the snowboard sales. The remaining part of the circle represents the fraction of the whole that is the snowshoe sales. This is graph B.

Incorrect Response A: This response could have come from seeing the differences in the three bars over Store 2 as equal and not relating all three parts to the whole (900), thus viewing the differences as three equal sections or thirds.

Incorrect Response C: This response could have come from misreading the graph and using sales from Store 3 instead of sales from Store 2.

Incorrect Response D: This response could have come from misreading the graph and using sales from Store 4 instead of sales from Store 2.

43. **Use the table below to answer the question that follows.**

Bread	**Number of Loaves Sold**
whole wheat	32
sesame seed	60
rosemary	25
poppy seed	18
sourdough	15

A bakery sells five kinds of bread. The table shows the number of loaves of each type of bread sold so far this week. Given the data in the table, what is the probability that the next loaf of bread sold will be sesame seed?

A. $37\frac{1}{2}\%$

B. 40%

C. 60%

D. $66\frac{2}{3}\%$

Correct Response B: Probability is expressed as the ratio of the number of ways to get a desired outcome compared to the number of ways to get all possible outcomes. In this case the number of all possible outcomes is the total number of loaves of bread sold so far in the week. The number of desired outcomes is the number of loaves of sesame seed bread sold so far. A total of 150 loaves of bread were sold so far this week (adding from the table). Out of the 150 loaves sold, 60 are sesame seed bread. 60 out of 150 is $\frac{60}{150} = \frac{6}{15} = \frac{2}{5}$. Two out of every five loaves of bread sold are sesame seed. To convert $\frac{2}{5}$ into a percent, the ratio must be represented by a ratio "out of 100": $\frac{2}{5} = \frac{40}{100} = 40\,\%$.

Incorrect Response A: This response could have come from making an addition error and finding the total number of loaves to be 160 instead of 150. 60 out of 160 = 37.5%.

Incorrect Response C: This response could have come from reading the 60 loaves of sesame seed bread given in the table and incorrectly assuming that it was 60% instead of the number of loaves.

Incorrect Response D: This response could have come from comparing the sales of sesame seed bread to the sales of all the other breads (150 – 60 = 90). 60 out of 90 is $\frac{2}{3}$, which is $66\frac{2}{3}$ %. The question asks for the probability of the next bread sold being sesame seed compared to all the bread sold, not to compare the wanted flavor of bread to all the unwanted flavors of bread.

44. A class kept track of the weather for 60 days and compared the actual weather to the local weather forecaster's predictions. Of the 15 days for which rain was predicted, there were 10 days of rain and 5 days with no rain. Of the 45 days for which no rain was predicted, there were 10 days of rain and 35 days with no rain. What is the probability that the forecast was accurate with regard to rain for any given day during the entire 60-day period?

    A. $\frac{1}{4}$

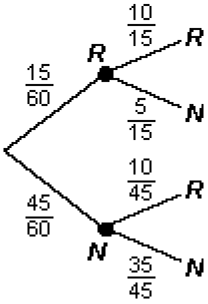
    B. $\frac{5}{12}$

    C. $\frac{1}{2}$

    D. $\frac{3}{4}$

Correct Response D: The most straightforward method of calculating the probability that the forecast was correct is to count the number of days when the forecast was correct and write a ratio of the number of correct predictions to the number of total predictions. There were 15 days of rain predicted and on 10 of those days it rained. There were 45 days of no rain predicted and on 35 of those days there was no rain. So, on 45 days (10 + 35) the forecast was correct. There were 60 days of predictions. There was a $\frac{45}{60} = \frac{3}{4}$ chance that the forecast was correct.

Alternatively, a tree diagram can be used to show the probabilities of each event. The first two branches represent the probability of rain or no rain based on the weather forecaster's predictions: probability of rain, *R*, is $\frac{15}{60}$ and probability of no rain, *N*, is $\frac{45}{60}$. The second set of branches show the probabilities based on actual occurrences of rain or no rain for the 15 days of rain and the 45 days of no rain predicted by the forecaster. The probability that the forecast was correct is the probability of *RR* plus the probability of *NN*. Since these events are independent of each other, the probability of *RR* is the product of the numbers on those branches: $\frac{15}{60} \bullet \frac{10}{15} = \frac{1}{6}$ and the probability of *NN* is the product of the numbers on those branches: $\frac{45}{60} \bullet \frac{35}{45} = \frac{7}{12}$. The probability that the forecast was accurate is $\frac{1}{6} + \frac{7}{12} = \frac{6}{36} + \frac{21}{36} = \frac{27}{36} = \frac{3}{4}$.



Incorrect Response A: This response could have come from misinterpreting "with regard to rain" and using the 15 days for which rain was predicted: $\frac{15}{60} = \frac{1}{4}$.

Incorrect Response B: This response could have come from using the tree to calculate the $\frac{1}{6}$ and $\frac{7}{12}$ as discussed above and then subtracting the fractions rather than adding them.

Incorrect Response C: This response could have been a guess, since none of the given data leads to a fraction of one-half.

45. A box contains 5 red cubes, 2 white cubes, and 7 purple cubes. One cube is randomly picked from the box. Without replacing the first cube, a second cube is randomly picked. What is the probability that *neither* of the cubes picked is purple?

    A. $\frac{1}{2}$

    B. $\frac{2}{7}$

    C. $\frac{3}{13}$

    D. $\frac{25}{26}$

Correct Response C: Probability is expressed as the ratio of the number of ways to get a desired outcome compared to the number of ways to get all possible outcomes. For the first draw, seven of the cubes are **not** purple, so the probability of choosing **non-purple** is 7 out of 14 cubes, or $\frac{7}{14} = \frac{1}{2}$. Assuming that the first cube picked was **not** purple, there are now 13 cubes to choose from in the second draw because one has already been removed and was not put back into the box. This means that there are 13 possible outcomes when one cube is drawn. Note that there are now only six **non-purple** cubes because one of those was chosen on the first draw. The probability of picking a **non-purple** cube will be 6 out of 13 possible choices, which is $\frac{6}{13}$. When the events are independent (i.e., picking a non-purple cube the first time does not determine what will be picked the second time), the probabilities of the two draws are multiplied: $\frac{7}{14} \bullet \frac{6}{13} = \frac{1}{2} \bullet \frac{6}{13} = \frac{6}{26} = \frac{3}{13}$.

Incorrect Response A: This response could have come from saying that there are 7 non-purple cubes out of a total of 14 cubes, which equals one-half.

Incorrect Response B: This response incorrectly compares the act of choosing 2 cubes to having 7 non-purple cubes from which to choose, resulting in 2 out of 7 or two-sevenths.

Incorrect Response D: This response comes from adding the two probabilities $\frac{1}{2} + \frac{6}{13} = \frac{13}{26} + \frac{12}{26} = \frac{25}{26}$ instead of multiplying them.