Massachusetts Tests for Educator Licensure®

GENERAL CURRICULUM MATHEMATICS SUBTEST

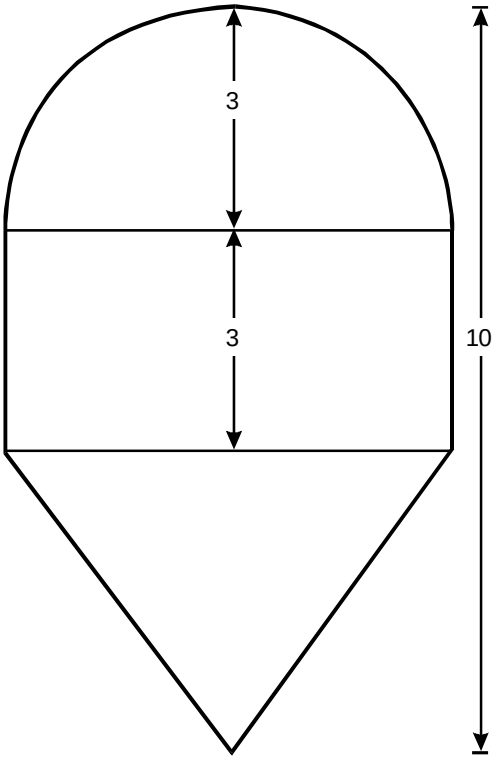
SUPPLEMENTAL SAMPLE OPEN-RESPONSE ITEM WITH SAMPLE RESPONSES AND ANALYSES

NOTE: This sample open-response item is provided as a supplement to the Test Information Guide and Practice Test provided for this test at www.mtel.nesinc.com.

SAMPLE OPEN-RESPONSE ITEM

Use the information below to complete the exercise that follows.

Elementary school students are asked to solve the following problem.



Approximately what fraction of the entire figure shown above is enclosed by the semicircle? (use $\pi = 3.14$)

Student response:

rectangle = 6 x 3 = 18

triangle = $\frac{1}{2}$ x 6 x 4 = 12

semicircle = $\frac{1}{2}$ x 2 x π x radius = 3π

3.14
x 3
93.12

18 + 12 + 93.12 = 123.12

answer = $\frac{93.12}{123.12}$ or about $\frac{90}{120} = \frac{3}{4}$

Use your knowledge of mathematics to create a response in which you analyze the student's work and provide an alternative solution to the problem. In your response, you should do the following:

- Correct any errors or misconceptions evident in the student's work.
- Explain why the response is not mathematically sound. Be sure to
  - provide a correct solution,
  - show your work, and
  - explain your reasoning.
- Solve the problem using an alternative method that could enhance the student's conceptual understanding of ratios and decimal multiplication in the context of the problem.

SAMPLE RESPONSES AND ANALYSES

Sample Score Point 4 Response

The process the student followed was correct. She had all the necessary steps. First, the areas of the rectangle and triangle were correct. She found "3" to be the radius of the semicircle, which means the diameter was "6." This was also the length of the rectangle. She used "4" as the height of the triangle, which was correct.

However, the formula for finding the area of the semicircle is incorrect. The student used the formula for the circumference ($2\pi r$) and divided by 2 to get half. She should have used $A = \pi r^2$ and found half of that number.

The student also made an error when multiplying $\pi \times 3$. She does not understand how to multiply decimals.

Correct solution:

$A = 3.14 \times 9$
$= 28.26 \times 1/2$
$= 14.13$ (not 93.12, which is what the student had)

So the area of the whole figure = 18 (area of rectangle) + 12 (area of triangle) + 14 (rounded area of semicircle) = 44

Therefore,

14/44 = 7/22 = approximately 1/3

I would review the formulas for finding the area and circumference of a circle with this student, so that she can use them correctly for problems like these. But for the multiplication mistake, I would talk with this student about how multiplying decimals is like multiplying fractions, and the decimal places represent fractional parts of a whole. I'd show her how to first estimate an answer by thinking of the whole numbers.

(continued on next page)

Sample Score Point 4 Response *(continued)*

Example: 12.5 x 10.3

The answer should be close to 120 because 12 x 10 is 120, while .5 and .3 are fractional parts of a whole-5 tenths and 3 tenths.

Then we could actually multiply 5/10 x 3/10 to show that the answer results in hundredth (15/100), which helps to explain how to figure out how many decimal places are needed in the answer when multiplying two or more decimals.

Then we could go back to the problem and try first estimating the answer to 3.14 x 3 (around 9), then do it by writing the decimal as a fraction (14/100) and multiplying it times 3/1, which equals 42/100.

Analysis of Sample Score Point 4 Response

Purpose: All aspects of this assignment have been fully addressed. The response provides a thorough analysis of each part of the student's work. That is followed by a correct, step-by-step problem-solving process and answer, as well as an alternative problem-solving approach that addresses a specific need the candidate identified to enhance this student's conceptual understanding of the rules for multiplying decimals.

Subject Matter Knowledge: The response is strengthened by the analysis of what the student did correctly as well as a discussion of where errors occurred in the student's work. By reviewing what the student did right, the candidate is demonstrating specific relevant mathematical knowledge, including the formulas for finding the areas of the rectangle and the triangle. The discussion of the errors (incorrect formula for finding the area of a semicircle and incorrect multiplication of decimals) and an alternative approach to decimal multiplication further reveal the candidate's substantial and accurate subject matter knowledge.

Support: The analysis of the student's work provides high-quality support that is often quite detailed, such as "She found '3' to be the radius of the semicircle, which means the diameter was '6.' This was also the length of the rectangle. She used '4' as the height of the triangle, which was correct." The candidate adds parenthetically what each number represents in the solution, e.g., "18 (area of rectangle)," support that strengthens the response overall. The alternative solution takes the reader through the explanation step-by-step with good detail.

Rationale: The candidate's depth of understanding is made clear by the manner in which this response deals with all aspects of the student's work. The candidate provides reasons why each step in the student's response is either correct or incorrect. The final section of the response, explaining where and why to focus on decimal multiplication with this student, and then showing a concrete example of how to do this, makes the candidate's reasoning process clear.

Sample Score Point 3 Response

The student has the correct areas for both the rectangle and the triangle. However, in the semicircle they should have used $A = \pi r^2$ and then taken half of it. The student also multiplied incorrectly.

Semicircle:

$A = .5 \times 3.14 \times 3^2$

$.5 \times 9.42$

$= 14.13$

$= 14/44 = 7/22 =$ about 1/3 of the figure

As an alternative method, I would review the formulas for these three figures. I would teach estimations of areas for geometric figures. But for the multiplication error, I would review the decimals. Then I would show them the difference between areas and perimeters.

Analysis of Sample Score Point 3 Response

Purpose: The candidate has largely addressed the assignment, identifying the errors that the student made, providing a correct solution, and suggesting an alternative method. However, there is only adequate explanation of why the student's work is not mathematically sound, and the alternative method explains only in general a "review" of the errors missed and does not solve the problem using the alternative method.

Subject Matter Knowledge: There is generally accurate application of math knowledge in the response. The candidate shows understanding of how to find the areas of three geometric figures (rectangle, triangle, and semicircle) and correct multiplication skills. The lack of a more elaborate explanation of the student's work means that the candidate is not able to show substantial math knowledge at the level of a "4" response. Without a fuller explanation of the alternative method, any math knowledge demonstrated remains general.

Support: The response provides adequate support by briefly identifying what the student has correct (areas of rectangle and triangle) and then giving the correct formula for a semicircle. Although the candidate solves correctly for the fraction, she leaves out steps that would fully explain her work. The alternative method is described only in general terms, without the examples that would make the response better than adequate.

Rationale: This is an adequately reasoned response. The candidate shows basic math knowledge and understanding of finding areas, multiplying decimals, and estimating fractions.

Sample Score Point 2 Response

The student should have used the area formula for the semicircle: $A = \pi r^2$.

Also, there was a multiplication error.

The areas for the rectangle and triangle were correct.

Therefore, 18 + 12 + 28 = 58

and 28/58 = 30/60 = 1/2

So it is 50% of the figure.

An alternative method would be to look at how big the semicircle is compared to the rest of the figure. 3/4 of the whole would be much larger than the size of the semicircle actually looks. The student could be shown that her answer is not logical just by looking at the figure.

Analysis of Sample Score Point 2 Response

Purpose: The purpose of the assignment is partially achieved. The two student errors are identified but not explained, the candidate's solution of the original problem is incorrect, and the proposed alternative method is more of a visual proof than a way to show the student how to approach the problem differently in order to enhance his conceptual understanding of ratios and decimals.

Subject Matter Knowledge: The math knowledge demonstrated is limited. The candidate knows the formulas for finding areas of rectangles, triangles, and semicircles, and understands that there was a multiplication error. But the response has no explanation of the student's work and does not demonstrate how to correctly multiply decimals. The suggested alternative method is limited by not presenting a genuinely different way of approaching the mathematical concepts involved with solving this problem.

Support: Support is limited since the problem is incorrectly solved and there is no explanation of the student's work or a viable alternative solution. The candidate does provide the formula for finding the area of a semicircle. There is no explanation of the steps that would detail how the candidate is solving the problem.

Rationale: There is limited understanding of the math involved in this assignment. There is little explanation of the student's errors, the method of solving the problem, or how the student might be helped to understand ratios and decimals by taking a different approach to the problem.

Sample Score Point 1 Response

The student's answer doesn't make any sense. Just looking at it you can tell it isn't 3/4 of the whole. First of all, the student did not multiply correctly. 3.14 x 3 = 9.32 = 10 rounded

So, the correct solution would be:

1/2 x 2 x 10 = 1 x 10 = 10

18 +12 + 10 = 40

10 / 40 = 1/4

Therefore, the semicircle is 1/4 of the whole figure.

Analysis of Sample Score Point 1 Response

Purpose: The purpose of the assignment is not achieved. Although the candidate does identify the multiplication error, he does not himself multiply correctly. The second error is not identified, there is no correct solution, and no alternative method is suggested.

Subject Matter Knowledge: There is a little math knowledge demonstrated in the multiplication of 3.14 × 3: although the answer is incorrect, it does reflect the correct placement of decimals. The solution to the original problem is wrong because the candidate does not know the correct formula for finding the area of a semicircle; he instead uses the student's formula for finding circumference.

Support: The support is weak. The few details present are not relevant because they are not solving the problem with the correct formula, explaining the student's misconceptions, or providing an alternative approach to understanding ratios and decimals.

Rationale: The response reflects little reasoning about the topic. The visual objection to the student's answer is acceptable, but there is little correct reasoning after that point. By the same reasoning, the candidate ought to be able to tell that his answer of ¼ is also incorrect.